Revealed Preferences of Individual Players in Sequential Games

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January 14, 2021

Abstract

This paper studies rational choice behavior of a player in sequential games of perfect and complete information without an assumption that the other players who join the same games are rational. The model of individually rational choice is defined through a decomposition of the behavioral norm assumed in the subgame perfect equilibria, and we propose a set of axioms on collective choice behavior that characterize the individual rationality obtained as such. As the choice of subgame perfect equilibrium paths is a special case where all players involved in the choice environment are each individually rational, the paper offers testable characterizations of both individual rationality and collective rationality in sequential games.

JEL Classification: C72, D70.
Keywords: Revealed preference, individual rationality, subgame perfect equilibrium.

1 Introduction

A game is a description of strategic interactions among players. The players involved in a game simultaneously or dynamically choose their actions, and their payoffs are determined by a profile of chosen actions. A number of solution concepts for games, such as the Nash equilibrium or the subgame perfect equilibrium, have been developed in the literature in order to study collective choice of actions made by the players. In turn, these solution concepts are widely applied in economic analysis to provide credible predictions for the choice of economic agents who reside in situations approximated by games.

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However, most of solution concepts for games are defined by preference relations (usually represented by payoff functions) of the players, while in practice only the choices of actions are observed. So, even when a certain solution concept appears reasonable to make predictions of the outcome of a game, we may not be able to apply such a concept unless preference relations of the players are known to the outside observer. Hence, it is important for the empirical contents of the game theory that there is a method that allows us to test the rationality of players and to reveal their preference relations on the basis of observed data set.

In this paper, by assuming observability of the choice of actions made by players involved in games, but not of their preference relations, we study a method to test whether they choose their actions rationally according to their preference relations. This question in principle follows the idea of the revealed preference theory pioneered by Arrow [1], Houthakker [8], Samuelson [14], Sen [16], and others. Yet, we depart from the classical revealed preference theory for an individual decision maker by assuming observability of *collective* choice behavior from games and by studying the rational choice behavior of multiple decision makers (or players) involved in such a choice environment. In this regard, Sprumont [17] takes the games of simultaneous moves and investigates axiomatizations of Pareto optimality and Nash equilibrium. Ray and Zhou [12] assume observability of paths chosen by the players in extensive games and characterize the choice of subgame perfect equilibria. In other contexts, Bossert and Sprumont [3] show that every choice function is rationalizable as a result of the backwards induction when the observed data is limited, Carvajal et al. [4] develop the revealed preference theory for Cournot competition games, and Carvajal and González [5] and Chiappori et al. [6] examine the testability of Nash solution in two-player bargaining games. Recently, Schenone [15] studies the relation between the subgame perfect equilibria and the backwards induction in a choice theoretic framework with, and without, the weak axiom of revealed preferences (WARP).

It is however worth noting that the existing literature has focused on behavioral characterizations of the *collective* rationality (by which we mean the game theoretic equilibria) in the collective choice environments, and a question of characterizing the *individual* rationality in this framework remains unanswered. Indeed, the historical and recent developments in experimental economics and in the theory of bounded rationality (Binmore et al. [2], Leng et al. [9], and others) suggest that decision makers tend to violate predictions of the rational choice theory in practice. Therefore, even when observed choice data does not support the collective rationality, it is still of interest to identify a set of players, if not all, who make individually rational choice. A question thus leads to finding a testable characterization of the individual rationality in the collective choice environment. This is what this paper is after. (See Table 1.)

To this end, the paper also relates to the work of Gul and Pesendorfer [7], in which they study intertemporal decision problems of a single decision maker with changing tastes. When the tastes of a decision maker change over time, the same decision maker may be tempted to deviate from a

	Individual rationality	Collective rationality
Individual choice data	Arrow [1] Houthakker [8] Samuelson [14] Sen [16]	n/a
Collective choice data	Gul and Pesendorfer [7] This paper	Bossert and Sprumont [3] Carvajal et al. [4] Carvajal and González [5] Chiappori et al. [6] Ray and Zhou [12] Schenone [15] Sprumont [17]

Table 1: Related literature

path of actions that she previously found optimal. In this situation, the model of consistent planning, originally proposed by Strotz [18], views the decision maker at different points in time as different players and solves for an action path that she can actually follow through. Gul and Pesendorfer axiomatize a representation termed the *weakly Strotz model*, in which only the decision maker at the initial period is assumed to be fully rational while she may, at any point in the future, choose her actions to maximize her payoffs given incorrect predictions of the beheavior of the subsequent selves. The present paper marks a contrast with the weakly Strotz model by characterizing the collective choice behavior where we postulate *no* behavioral assumption on players at any decision nodes in the future. To highlight a difference, for any game with 2 periods, the weakly Strotz model coincides with the evaluation of the game by the optimal subgame perfect equilibrium path, as player 2, the player at the last period, does not have to form any predictions for the subsequent players and is hence rational. In contrast, the concept of the individual rationality studied in this paper allows behavioral choice of the players at period 2 (see Example 4).

This paper aims to characterize the individual rationality in the collective choice environment, without relying on any behavioral assumption on the other players. In particular, following Gul and Pesendorfer [7] and Ray and Zhou [12], we adopt a plain setup of sequential games where linearly ordered players make their actions one after another, and observability is assumed for paths of actions chosen by the players in such games. We focus on a person, called player 1, who chooses an action first among all the players (i.e. the one who stands at the initial node) and seek a testable axiomatization of the individual rationality for this player. A notion of individual rationality in our simple framework is derived from a decomposition of the behavioral norm assumed in the subgame perfect equilibria into the level of the individual player. Specifically, if player 1 is rational, then she should correctly form a history-dependent prediction of actions chosen by the subsequent players and choose her own action in order to achieve an optimal path among those that are actually followed by the players. Importantly, the other players may choose *unrationalizable* actions, but even then, player 1 takes such into consideration and chooses her action to achieve the best outcome among those that she can achieve. The main theorem of the paper provides an axiomatization on the observed choice data that is equivalent to the rationality of player 1 in this sense.[1]

The collective rationality is reconstructed from the individual rationality. We show that any path chosen by the players in sequential games must be a subgame perfect equilibrium path, provided that *all* players in the environment is individually rational in the same sense above. Notably, since the individual rationality of each player is testable through a plain adjustment of the same axiomatization for player 1, this paper thus provides an axiomatic characterization for the choice of subgame perfect equilibrium paths in sequential games, parallel to the result by Ray and Zhou [12].[2] We will also demonstrate a subtle yet important difference between the collectively rational choice and the choice of subgame perfect equilibria.

The paper is structured as follows. In Section 2, we introduce a model of sequential games used throughout the paper and then place the main concept of this paper, the individually rational choice correspondences. We will also use some examples to motivate the model. In Section 3, we discuss certain testable axioms on observed choice behavior in sequential games and show that the proposed axioms characterize the individual rationality in the sequential games. The collective rationality is studied in Section 4. We show that we can use the same set of axioms to test whether the observed choice behavior is collectively rational, and, moreover, that a collectively rational choice correspondence closely relates to the choice of subgame perfect equilibria. All proofs are given in the appendix.

2 Individually rational choice in sequential games

2.1 Rationalizable choice

Let X be any nonempty set, and $\mathcal{X}$ a collection of nonempty subsets of X. A choice correspondence on $\mathcal{X}$ is a map $C : \mathcal{X} \to 2^X$ such that $\emptyset \neq C(S) \subseteq S$ for every $S \in \mathcal{X}$. When X represents a set of conceivably all alternatives in interest, a choice correspondence describes a decision maker's

[1] Of course, there are other ways to formulate a model of individual rationality. For example, if we instead decompose the solution concept by Pearce [11], we would obtain a weaker notion in which player 1 may have uncoordinated predictions for the choice of actions by the other players. Providing revealed preference tests for such alternative models of individual rationality is beyond the scope of this paper.

[2] Indeed, this paper generalizes an axiomatic characterization by Ray and Zhou [12] by allowing possibly set-valued choice observations and game trees of infinite horizon.

choice behavior on $\mathcal{X}$. In particular, we interpret that, if $S \in \mathcal{X}$ is a set of feasible alternatives, the decision maker is willing to choose any alternative in $C(S)$ from S. With this interpretation, the collection $\mathcal{X}$ hence consists of *choice sets* from which we observe the decision maker's choice behavior. A choice correspondence is often interested when it is in a certain manner associated with a preference relation of the decision maker. Throughout the paper, *a preference relation* on X refers to a complete and transitive binary relation on X. In turn, we say that a choice correspondence C on $\mathcal{X}$ is said to be *rationalizable* if there is a preference relation $\succsim$ on X such that $C(S) = \{x \in S : x \succsim y \text{ for all } y \in S\}$ for every $S \in \mathcal{X}$.

2.2 Sequential games

In this paper, we study collective choice of paths of actions from sequential games. More specifically, we focus on sequential games of perfect information where players $t = 1, 2, 3, \ldots$ are linearly ordered and sequentially choose their actions with the knowledge of the actions chosen by the preceding players. Importantly, we only assume observability of chosen paths in the sequential games and not of the players' preference relations (or payoff functions) over the paths. We instead postulate that the unobserved preference relations of the players dictate their decision making, and, in turn, we attempt to reveal their preferences from the observed collective choice behavior.

In order to describe such a choice environment, let A be an arbitrary nonempty set and X be a nonempty set of infinite sequences of members of A. In this paper, we refer to any member of A as *an action* and to that of X as *a path* of actions. For any path $p \in X$ and $t \geq 1$, we write p^t to denote the first t terms of p, that is, $p^t = (p_1, \ldots, p_t)$. We interpret that the set X consists of *conceivably all* paths that the players may follow without any feasibility constraint. However, the players may face some restrictions on the feasible paths, and this restriction is represented by a subset G of X that consists of only available paths for their choice. Now, a sequential game is defined as a nonempty subset G of X with a certain closedness property.

Definition. A sequential game (or *a game* for short) in X is a nonempty subset G of X such that, for any infinite sequence p of actions in A, if, for every $t \geq 1$, $q^t = p^t$ for some $q \in G$, then $p \in G$.

It is straightforward from the definition to show that any singleton subset of X is a sequential game and that any union of two sequential games is also a sequential game. The closedness property essentially makes the definition of sequential games by paths of actions adopted in this paper equivalent with the definition of sequential games by nodes and feasible actions at each node. The next example further clarifies this property.

Example 1 (Tree cutting problem)**.** To interpret the definition of sequential games, let $A = \{0, 1\}$, and $X = \{\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \ldots\}$, where $\mathbf{0}$ denotes the infinite sequence of zeros, and $\mathbf{e}_t$ is the tth unit vector for each $t \geq 1$. (See Figure 1.) In association with the tree cutting problem, we interpret $\mathbf{0}$ as a path

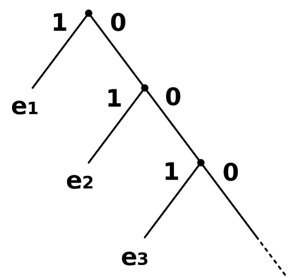

Figure 1: A tree cutting problem

where the tree is never cut and $\mathbf{e}_t$ as one where the tree is cut by player t. Now, let G be a subset of X such that $\{\mathbf{e}_1, \mathbf{e}_2, \ldots\} \subseteq G$, and suppose that G is a sequential game. In this game, since $\mathbf{e}_{t+1} \in G$, every player $t \geq 1$ can choose *not* to cut the tree provided that it has not been cut by the preceding players. But, then, the path $\mathbf{0}$, in which all the players choose not to cut the tree, should be also a feasible path in this game. Indeed, since $\mathbf{e}^t_{t+1} = \mathbf{0}^t$ for all $t \in \mathbb{N}$, the definition above implies that $\mathbf{0} \in G$.

Remark (Games with finite horizon)**.** Though we assume that every path in X is an infinite sequence of actions, the present framework imposes no loss of generality with respect to the number of players. An environment that involves only finitely many players can be formulated by augmenting players each of whom has the only one action $\emptyset$, where $\emptyset$ is an arbitrary fixed member of A, interpreted as "do nothing." Formally, if the collection of conceivably all paths is given by a set Y of sequences in A of finite length, we can proceed the analysis by setting $X := \{(p, \emptyset, \emptyset, \ldots) : p \in Y\}$. Similarly, the present framework allows sequential games that terminate at different periods depending on paths of actions as in the next example.

Example 2 (Dynamic Bertrand competition with entry regulation)**.** Suppose that, in a certain market, firms are engaged in dynamic Bertrand competition under an entry regulation.[3] We interpret that players $t = 2, 3, \ldots$ are firms who wish to enter the market, whereas player 1 is a regulator of the market who decides the number of the firms $n \in \{1, 2, \ldots\}$ that can actually enter the market. We assume that the firms are aligned in the order of priority to enter the market, and those who entered the market can manifest prices of their products in the same order. Then, this environment can be modeled by a set $A = \mathbb{N} \cup P \cup \{\emptyset\}$ of actions, where P is a set of prices chosen by the firms, and a set

$$X = \bigcup_{n \in \mathbb{N}} (\{n\} \times P^n \times \{\emptyset\}^\infty)$$

[3]In particular, small markets such as a farmers market or a comic market may fit well in the context of this example.

of conceivably all paths for the players to follow. In particular, a sequential game in this environment effectively terminates at period $n + 1$ when player 1 chooses an action n in the first period.

Let $\mathcal{G}$ be the collection of all sequential games in X. For any game $G \in \mathcal{G}$, we define $H_1(G) = \{\emptyset\}$ and $H_t(G) = \{p^{t-1} : p \in G\}$ for each $t \geq 2$. Then, denote $H(G) = \bigcup_{t\geq 1} H_t(G)$. We refer to any member of $H_t(G)$ as *a history* at period t and any member of $H(G)$ as a history in G.[4] In turn, we define $G_h := \{p \in G : p^{t-1} = h\}$ for any game $G \in \mathcal{G}$, any period $t \geq 1$, and any history $h \in H_t(G)$. Here, G_h is a game in $\mathcal{G}$ in which the players play the game G while the first $t-1$ players commit to follow the history h. In this paper, we identify the game G_h with the continuation game in G after the history h.[5] We postulate that, once the players face a sequential game G in $\mathcal{G}$, they collectively choose a path of actions in the following manner. First, player 1 chooses its action a_1. The action a_1 must be feasible in the sense that there is a path $p \in G$ with $p_1 = a_1$. (Or, equivalently, $a_1 \in H(G)$.) Next, player 2 chooses its action a_2, knowing that player 1 chose the action a_1. Player 2's action has to be feasible after a history a_1 in the sense that $(a_1, a_2) \in H(G)$. Inductively, for every $t \geq 3$, and for every history $h \in H(G)$ of length $t - 1$, provided that the players up to $t - 1$ have chosen their actions $h = (a_1, \ldots, a_{t-1})$, player t chooses its feasible action a_t such that $(h, a_t) \in H(G)$. As a result of this procedure, a path $q = (a_1, a_2, \ldots)$ of actions is chosen by the players in the game G, where the definition of the sequential games guarantees that $q \in G$. The players are responsible for the choice of their own actions, but they have no control for the actions of the other players. We assume observability of the collective choice of paths made in this way from each sequential game in $\mathcal{G}$. In other words, the paper takes a choice correspondence C on $\mathcal{G}$ as a primitive of the model.

Remark. Important remarks regarding the assumption of the choice observation are in order.

(i) All of the results of this paper remain true if $\mathcal{G}$ is a nonempty collection of sequential games in X such that $\mathcal{G}$ contains all singletons of X, $\mathcal{G}$ is closed under taking unions, and $\mathcal{G}$ is closed under taking continuation games (that is, $G \in \mathcal{G}$ and $h \in H(G)$ imply $G_h \in \mathcal{G}$). While the collection of all sequential games in X taken above for simplicity is an instance of $\mathcal{G}$ that satisfies these conditions, we could alternatively take $\mathcal{G}$ as, for example, the collection of all *finite* subsets of X. The latter framework can be useful in experimental contexts, for it allows us to test the individual rationality of the players only on the basis of the observation of collective choice behavior from finitely many paths.

[4]The unique member of $H_1(G)$ represents the null history. In what follows, with abuse of notation, we identify any history of length 0 with the null history $\emptyset$ and any history $h = (a)$ of length 1 with the action a itself. Therefore, the set $H_2(G)$ of histories at period 2 coincides with the set of feasible actions for player 1 in the game G.

[5]This entails a behavioral assumption that the actions actually chosen by the preceding players can affect, but the presence of the other feasible actions that they could have chosen does not affect, the choice of actions by the subsequent players. One justification of this assumption is that the subsequent players may only observe the chosen actions by the preceding players, but not the feasible sets of actions for them.

(ii) By taking a choice correspondence C on $\mathcal{G}$ as the primitive of the model, though we do not assume the observability of the players' preference relations, the structures of game trees and the choice of paths from each game tree are assumed to be observable. In contrast, Bossert and Sprumont [3] and Rehbeck [13] show that *any* choice correspondence is backwards induction rationalizable if we only observe chosen outcomes of the games but not the game trees. The assumption of the observability of the game structures is hence indispensable for refutability of the models of rational choice studied in this paper.

(iii) An implicit assumption adopted throughout the paper is that the players involved in the choice environment choose pure strategies. A natural extension of interest would be to consider an environment where the players may choose mixed strategies and we observe a stochastic choice function that associates each sequential game G in $\mathcal{G}$ with a probability distribution (or a set of probability distributions) over the paths in G. The axiomatic characterizations of individually rational and collectively rational choice behavior in such a stochastic choice environment are left as an open question.

2.3 Individual rationality

We seek a testable characterization of individually rational choice behavior by player 1 independent of any behavioral assumptions on the other players. We define such a model of rational choice through two requirements. The first requirement is the rational prediction of player 1 for the behavior of the other players. Suppose that the players face a game G in $\mathcal{G}$. In this game, the set $H_2(G)$ of histories at period 2 coincides with the set of feasible actions for player 1. (See footnote 4.) Then, by playing a feasible action $a \in H_2(G)$, player 1 passes the continuation game G_a to the subsequent players. Note that the players are observed to choose paths in $C(G_a)$ from the continuation game G_a. If player 1 correctly anticipates the behavior of the other players, then she should not choose an action a in order to achieve a path $p \notin C(G_a)$ which is not followed through by the subsequent players. The first condition of individual rationality requires that player 1 expect the other players to follow a path in $C(G_a)$ after each of her feasible action $a \in H_2(G)$ at the initial node. Notably, we postulate that player 1 expects a path *actually chosen* by the other players in the continuation game G_a to follow after any feasible action $a \in H_2(G)$ of herself. This assumption sets our model of individually rational choice to respect the idea of the subgame perfect equilibrium and to distinguish itself from that of the Nash equilibrium.

The second requirement of the individual rationality is that player 1 chooses her action in order to achieve an optimal path given the belief for the behavior of the other players. To be specific, we postulate that player 1 has a preference relation on the set X of paths, and, for any $a \in H_2(G)$, she prefers a chosen path $p \in C(G)$ to a path $q \in G_a$ that she believes the other players to follow in case she plays an alternative action a. Below we show that this requirement naturally extends the traditional concept of individual rationality, that is, the model of preference maximization, to the present framework (Example 3). These two requirements lead to the following definition of what we term *individually rational* choice behavior in the collective choice environment.

Definition. A choice correspondence C on $\mathcal{G}$ is individually rational (at the initial node) if there exists a preference relation $\succsim$ on X such that, for all $G \in \mathcal{G}$, the following two statements are equivalent:

(a) $p \in C(G)$.

(b) $p \in C(G_{p_1})$, and for any $a \in H_2(G)$, there exists a $q \in C(G_a)$ with $p \succsim q$.

Example 1 (continued)**.** In the tree cutting problem, suppose that cutting the tree is a burdensome but urgent matter for every player. Specifically, where $\succsim_t$ denotes a preference relation of player t, let $\mathbf{e}_{t+1} \succ_t \mathbf{e}_t \succ_t \mathbf{e}_s \succ_t \mathbf{0}$ for all $s > t + 1$. (Therefore, every player prefers that the next player cuts the tree, but she would rather prefer to do so by herself if any subsequent player cuts the tree.) In addition, assume that player t, for every $t \geq 2$, is myopic in the sense that she does not cut the tree as long as it is feasible for the next player to do so. In this environment, if player 1 makes its choice rationally, the collective choice behavior will be described by a choice correspondence C such that

$$C(G) = \begin{cases} \mathbf{e}_1 & \text{if } \mathbf{e}_1, \mathbf{e}_3 \in G, \\ \mathbf{e}_{t^*} & \text{otherwise} \end{cases} \qquad \text{for any } G \in \mathcal{G},$$

where t^* is the first t such that $\mathbf{e}_t \in G$ and $\mathbf{e}_{t+1} \notin G$ (or $\mathbf{e}_{t^*} = \mathbf{0}$ if no such t exists). In particular, if $\mathbf{e}_3 \in G$, player 1 cuts the tree whenever possible, rationally expecting that player 2 will not do so. In contrast, any subsequent players are only boundedly rational, possibly ending up with the path $\mathbf{0}$ even when it is feasible for them to cut the tree. To be specific, $C(G) = \mathbf{0}$ when $G = \{\mathbf{0}, \mathbf{e}_2, \mathbf{e}_3, \ldots\}$, which player Notice that C is therefore not rationalizable The choice correspondence C is individually rational at the initial node.

Example 3 (Rationalizable choice)**.** It is straightforward to show that any rationalizable choice correspondence on $\mathcal{G}$ is individually rational at the initial node. (In fact, under the jargon introduced in Section 4, any rationalizable choice correspondence is individually rational at all decision nodes and hence collectively rational.) The converse is yet false in general. Indeed, even when all players involved in the choice environment is individually rational, the observed choice behavior C might not be rationalizable. This is because, in the current collective choice environment, each player only makes choice of her own action and cannot control those of the subsequent players, and consequently, she may fail to achieve her most preferred path in a game.[6] Now, let X be a set of

[6] For a concrete example, let $A = \{0, 1, x, y, \emptyset\}$ and $X = \{\mathbf{0}, \mathbf{x}, \mathbf{y}\}$, where $\mathbf{0} = (0, \emptyset, \ldots)$, $\mathbf{x} = (1, x, \emptyset, \ldots)$, $\mathbf{y} = (1, y, \emptyset, \ldots)$.

paths of form $p = (a, \emptyset, \emptyset, \ldots)$ for some $a \in A$, where $\emptyset$ is an arbitrary fixed member of A. We interpret X as a choice environment for a single agent. In this environment, a choice correspondence C on $\mathcal{G}$ is individually rational if and only if it is rationalizable. Hence, the model of individually rational choice behavior introduced above coincides with the standard notion of rationalizable choice correspondences when there is only one player who makes a choice.

Example 4 (Choice under the attraction effect)**.** Consider an environment where player 1, a seller, chooses whether to enter a market or not, and, if the seller enters, then player 2, a buyer, chooses a product to purchase from up to three alternatives x, y, and z. We interpret that x is the product of the seller whereas y is that of a competitor, and it is profitable for this seller to enter the market only if the buyer purchases x. The buyer is, however, boundedly rational and chooses x over y only when z is feasible. (The alternative z is a decoy and never chosen by the buyer unless necessary.) Suppose that the seller is rational and correctly anticipates the buyer's behavior. To describe this choice environment, let $A = \{0, 1, x, y, z, \emptyset\}$ and $X = \{\mathbf{0}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$, where $\mathbf{0} = (0, \emptyset, \ldots)$ and $\mathbf{a} = (1, a, \emptyset, \ldots)$ for each $a = x, y, z$. (See Figure 2.) Then, the choice behavior of the players above is described by a choice correspondence C such that

$$C(G) = \begin{cases} \mathbf{x} & \text{if } G = X, \{\mathbf{0}, \mathbf{x}, \mathbf{z}\}, \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}, \text{or } \{\mathbf{x}, \mathbf{z}\} \\ \mathbf{y} & \text{if } G = \{\mathbf{x}, \mathbf{y}\} \text{ or } \{\mathbf{y}, \mathbf{z}\} \\ \mathbf{0} & \text{if } G = \{\mathbf{0}, \mathbf{x}, \mathbf{y}\} \text{ or } \{\mathbf{0}, \mathbf{y}, \mathbf{z}\} \end{cases} \qquad \text{for any } G \in \mathcal{G}.$$

In particular, player 1 chooses to enter the market when $G = X$ (as z is present), while it defers to do so when $G = \{\mathbf{0}, \mathbf{x}, \mathbf{y}\}$. Importantly, player 2's choice behavior in the games cannot be described by the standard model of preference maximization. The choice correspondence C is nevertheless individually rational at the initial node.

3 Axiomatic characterization of individual rationality

In the last section, we introduced the model of individual rationality in sequential games. We shall seek a characterization of this model by testable axioms in what follows. To this end, it will be convenient to write

$$\Gamma(G) \coloneqq \{G_a : a \in H_2(G)\}$$

Then, a choice correspondence C on $\mathcal{G}$ defined by

$$C(\{\mathbf{0}, \mathbf{x}\}) = \{\mathbf{x}\}, \quad C(\{\mathbf{0}, \mathbf{y}\}) = \{\mathbf{0}\}, \quad C(\{\mathbf{x}, \mathbf{y}\}) = \{\mathbf{y}\}, \quad C(\{\mathbf{0}, \mathbf{x}, \mathbf{y}\}) = \{\mathbf{0}\}$$

is individually rational at the initial node. In fact, it is collectively rational under the term introduced in Section 4. (Note that C is the choice of the subgame perfect equilibrium paths, where the preference relations of player 1 and player 2 are such that $\mathbf{x} \succ_1 \mathbf{0} \succ_1 \mathbf{y}$ and $\mathbf{y} \succ_2 \mathbf{x}$.) The choice correspondence C is not rationalizable, for $C(\{\mathbf{0}, \mathbf{x}\}) = \{\mathbf{x}\}$ whereas $C(\{\mathbf{0}, \mathbf{x}, \mathbf{y}\}) = \{\mathbf{0}\}$. Player 1 fails to achieve her favorite path $\mathbf{x}$ even when it is feasible in the game $\{\mathbf{0}, \mathbf{x}, \mathbf{y}\}$.

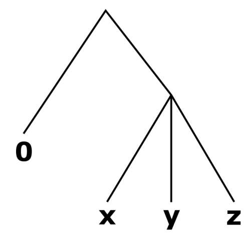

Figure 2: Choice under the attraction effect

for any game G in $\mathcal{G}$. This is the decomposition of the game G into its continuation games which player 1 can pass to the subsequent players. Note that $\bigcup \Gamma(G) = G$, and any distinct members of $\Gamma(G)$ are pairwise disjoint, and hence $\Gamma(G)$ is a partition of G.

The model of the individual rationality in Section 2.3 is characterized via three levels of axiomatization. The first level of the axiomatization is a requirement that, without altering the structures of continuation games, the choice of an action by player 1 (to achieve a certain path) be pairwise optimal against the other actions of player 1. Suppose that G is an arbitrary game, p is a feasible path in G, and $a \in H_2(G)$ is an arbitrary action of player 1 in G. Then, consider a game $G_{p_1} \cup G_a$, in which player 1 can choose either the action p_1 to pass the continuation game G_{p_1} or the action a to pass the continuation game G_a. If we observe that the path p is chosen from this game, that is, $p \in C(G_{p_1} \cup G_a)$, then player 1 finds it acceptable to choose the action p_1 to achieve the path p when it is alternatively feasible to take the action a. Therefore, the choice of the path p is in this sense "supported" by player 1 against the continuation game G_a. The first axiom requires that the path p be chosen from the game G if, and only if, it is supported by player 1 in the same sense above for *every* pairwise comparison with continuation games of the original game G.

A1. For any $G \in \mathcal{G}$ and $p \in G$, $p \in C(G)$ if and only if $p \in C(G_{p_1} \cup G')$ for all $G' \in \Gamma(G)$.

Note that, under A1, a choice correspondence is fully characterized by the choice from games in which player 1 has at most two actions to take. The second level of the axiomatization then imposes a condition on the collective choice behavior for such games. In particular, the next axiom requires that, if player 1 has exactly two actions in a game, then the choice from the game coincides with the removal of dominated paths against those that player 1 anticipates to result when she chooses alternative actions. To illustrate, suppose that G is an arbitrary game such that $\Gamma(G) = \{G_a, G_b\}$ with $G_a \neq G_b$. In this game, player 1 has two distinct actions a and b, and the continuation games are given by G_a and G_b, respectively. Also, suppose that $p \in C(G_a)$, and hence a path p is chosen by the subsequent players in the continuation game G_a. Then, under what condition would it be reasonable that we observe the same path p *not* chosen from the game G? Note that player 1 can avoid the choice of the path p in the game G by playing the action b. If she chooses the action b

and passes the continuation game G_b to the subsequent players, player 1 can anticipate that a path from $C(G_b)$ will take place. Now, let $q \in C(G_b)$ be such a path, and suppose that $C(\{p,q\}) = \{q\}$. Observe that $p_1 = a \neq b = q_1$, and hence, in the game $\{p,q\}$, player 1 can unilaterally commit to the choice of a path (by choosing the action a or b). Therefore, we infer from the condition $C(\{p,q\}) = \{q\}$ that player 1 prefers q over p. If this holds for any path q in $C(G_b)$, then player 1 has no reason not to choose the action b to avoid the path p. The second axiom requires that the path p be not chosen from the game G if, and only if, player 1 prefers *every* path in $C(G_b)$ to the path p in the binary comparison.

A2. For any distinct G and G' in $\mathcal{G}$ with $\Gamma(G \cup G') = \{G, G'\}$ and $p \in C(G)$, $p \notin C(G \cup G')$ if and only if $C(\{p,q\}) = \{q\}$ for all $q \in C(G')$.

Under A1 and A2, a choice correspondence boils down to be determined by the choice behavior $C(\{p,q\})$ on binary comparisons (besides the choice behavior on the continuation games). The last level of the axiomatization then imposes a consistency condition for these binary choice observations. To facilitate the exposition, we shall say that a path p is *revealed preferred to* another path q if $p_1 \neq q_1$ and $p \in C(\{p,q\})$. This construction follows the idea that the observed choice behavior reveals a preference relation of player 1 when she can commit to the choice of a path in a game, as already used in the justification of A2. Then, we shall in turn say that a path p is *indirectly revealed preferred to* another path q if there exists a path r such that $p_1 \neq r_1 \neq q_1$, $p \in C(\{p,r\})$ and $r \in C(\{r,q\})$. This construction allows the inference of the preference relation of player 1 for two paths p and q even when $p_1 = q_1$. Now, suppose that p is indirectly revealed preferred to q, and $q \in C(\{q\} \cup G)$ where G is a game such that $\{q\} \in \Gamma(\{q\} \cup G)$. The last condition, $\{q\} \in \Gamma(\{q\} \cup G)$, implies that player 1 can commit to the choice of the path q in a game $\{q\} \cup G$. Therefore, here we assume that player 1 is willing to commit to the choice of the path q in the game where she can do so. The last axiom then requires that, by replacing the path q with the preferred path p, if player 1 could still commit to the choice of the path p, then she be willing to do so.

A3. Let $p, q \in X$ and $G \in \mathcal{G}$ be such that $\{p\} \in \Gamma(\{p\} \cup G)$ and $\{q\} \in \Gamma(\{q\} \cup G)$, and suppose that $p \in C(\{p,r\})$ and $r \in C(\{r,q\})$ for some $r \in X$ with $p_1 \neq r_1 \neq q_1$. Then, $q \in C(\{q\} \cup G)$ implies $p \in C(\{p\} \cup G)$.

We show that the three axioms above characterize our model of the individual rationality.

Theorem 1. *A choice correspondence C on $\mathcal{G}$ is individually rational at the initial node if, and only if, it satisfies A1, A2, and A3.*

By Theorem 1, if a choice correspondence C satisfies A1 through A3, then we know that there exists a preference relation $\succsim$ on X under which C is individually rational at the initial node. In fact, we can show that the revealed preference relation introduced in the discussion of the axioms above provides us with the unique part of rationalizing preference relations. To formally state the result, given an arbitrary choice correspondence C on $\mathcal{G}$, we shall define a binary relation $\succsim_C^\emptyset$ on X (with its strict part $\succ_C^\emptyset$) by $p \succsim_C^\emptyset q$ iff (i) $p = q$, (ii) p is revealed preferred to q, or (iii) p is indirectly revealed preferred to q. Importantly, the binary relation $\succsim_C^\emptyset$ is constructed only on the basis of the observed choice correspondence C and hence it is itself observable. The next proposition follows.

Proposition 2. *Suppose that a choice correspondence C on $\mathcal{G}$ is individually rational at the initial node. Then, $\succsim_C^\emptyset$ is a preorder on X, and the two statements below are equivalent for any preference relation $\succsim$ on X.*

(i) C is individually rational at the initial node under $\succsim$.

(ii) $\succsim$ extends $\succsim_C^\emptyset$.

Moreover, given any preorder $\trianglerighteq$ on X (with its strict part $\triangleright$), there exists a preference relation $\succsim$ on X such that $\succsim$ extends $\trianglerighteq$ and C is individually rational at the initial node under $\succsim$, provided that

$$p \; tran(\succsim_C^\emptyset \cup \trianglerighteq) \; q \quad \Longrightarrow \quad neither \;\; q \succ_C^\emptyset p \;\; nor \;\; q \triangleright p \;\; holds \tag{1}$$

for any $p, q \in X$.[7]

The first half of Proposition 2 provides the uniqueness result on rationalizing preference relations for the model of individually rational choice behavior. Specifically, given a choice correspondence C on $\mathcal{G}$ that is individually rational at the initial node, it shows that any preference relation $\succsim$ extending $\succsim_C^\emptyset$ must rationalize C, and, conversely, any preference relation $\succsim$ rationalizing C must extend $\succsim_C^\emptyset$. The second half of Proposition 2 offers a testable condition for the existence of a rationalizing preference relation $\succsim$ that extends a given preorder on X. This result is particularly useful when we are given exogenous knowledge about a preference relation of the player and interested in testing consistency of the observed choice behavior against such knowledge. (A similar condition is used in the context of consumer revealed preference theory in Nishimura et al. [10, Proposition 1].)

Remark. Important remarks on Proposition 2 are in order.

(i) Note that $\succsim_C^\emptyset$ is constructed only by the choice observation of form $C(\{p, q\})$. Therefore, while we need the observation from richer choice problems in order to test the individual rationality of a player in our framework, the exercise of revealed preference itself can be conducted with only the choice observation from *pairwise* choice problems. This fact could serve in handy in experimental contexts to design an efficient experiment for revealing preference relations of the subjects.

[7] In this proposition and what follows, given any two binary relation R and P on X (with the corresponding strict parts $R^\succ$ and $P^\succ$), we say that R extends P if $R \supseteq P$ and $R^\succ \supseteq P^\succ$. Also, we shall denote by tran(R) the transitive closure of R.

(ii) Throughout the paper, we assume that the players have preference relations over the set X of paths of actions. However, it is sometimes assumed that each path of actions is mapped to a certain outcome, and the players have preference over the outcomes rather than over the paths of actions. The individual rationality of the observed choice behavior in such a framework can be tested with a requirement (in addition to A1-A3) that the player must be indifferent for any two paths that lead to the same outcome. To formally state the result, let Z be a nonempty set of outcomes, and suppose that each path in X leads to an outcome in Z according to a mapping $\xi : X \to Z$. Then, by defining an equivalent relation $\approx$ on X by $p \approx q$ iff $\xi(p) = \xi(q)$, we seek a testable condition under which the observed choice C is individually rational at the initial node under a preference relation $\succsim$ that extends $\approx$. By Proposition 2, this condition can be written as

$$p \ \mathrm{tran}(\succsim_C^{\emptyset} \cup \approx) \ q \quad \Longrightarrow \quad \text{not} \ \ q \succ_C^{\emptyset} p$$

for any $p, q \in X$.

4 Collective rationality in sequential games

4.1 Individually rational choice at an arbitrary decision node

In Section 2.3, we introduced the model of individually rational choice in sequential games. There we postulate that player 1, a player who makes decision at the initial node, is rational, and we study a representation in which the player makes a rational choice of her action in order to achieve an optimal path given the choice behavior of the subsequent players. This section extends the concept of the individual rationality for a player who stands at an arbitrary decision node. We shall write

$$H := \bigcup_{G \in \mathcal{G}} H(G) = \{p^{t-1} : p \in X, t \geq 1\}.$$

This is the set of all histories across all games in $\mathcal{G}$. Given that this paper studies sequential games of perfect information, we may identify an arbitrary history in H with a decision node of a player who makes choice given the same history. In this sense, with abuse of terminology, we will refer to any history in H also as *a decision node*. Let $h \in H$ be a decision node. Then, for an arbitrary game $G \in \mathcal{G}$, there may or may not exist a path $p \in G$ that "passes through" this decision node h. We have $h \in H(G)$ if such a path exists and $h \notin H(G)$ otherwise. When $h \in H(G)$, the player who makes choice at the decision node h faces the continuation game G_h, provided that a play of the game reaches the decision node h. At this node, an action $a \in A$ is feasible if and only if $(h, a) \in H(G)$, and she passes the continuation game $G_{(h,a)}$ to the subsequent player by choosing a feasible action a. The next definition gives a straightforward extension of the individually rational choice correspondences introduced in Section 2.3 for arbitrary decision nodes. (Indeed, it reduces to the definition in Section 2.3 when h is the null history $\emptyset$.)

Definition. Given an arbitrary $h \in H$, a choice correspondence C on $\mathcal{G}$ is individually rational at a decision node h if there exists a preference relation $\succsim$ on X such that, for all $G \in \mathcal{G}$ with $h \in H(G)$, the following two statements are equivalent:

(a) $p \in C(G_h)$.

(b) $p \in C(G_{(h,p_t)})$, and for any a with $(h, a) \in H(G)$, there exists a $q \in C(G_{(h,a)})$ with $p \succsim q$,

where $t \geq 1$ in the second statement is such that $p^{t-1} = h$ (so $t - 1$ equals the length of h).

In Section 3, we provide an axiomatic characterization of the individually rational choice correspondences at the initial node. Since the individual rationality at an arbitrary decision node h is defined by the same representation only relative to the node h, we can indeed characterize it with an application of the same set of the axioms. To see this, let C be a choice correspondence on $\mathcal{G}$, and fix any decision node $h \in H$. We set $X|_h := \{p : (h, p) \in X\}$ and let $\mathcal{G}|_h$ be the set of all sequential games in $X|_h$. Then, define a choice correspondence $C|_h$ on $\mathcal{G}_h$ by

$$C|_h(G) = \{p : (h, p) \in C(\{(h, q) : q \in G\})\} \tag{2}$$

for any $G \in \mathcal{G}|_h$. Note that the construction $(X|_h, \mathcal{G}|_h, C|_h)$ extracts the choice environment and choice observations at and after the history h from the original $(X, \mathcal{G}, C)$ while "shifting" the decision node h to the initial node. Hence, the individual rationality of the choice correspondence C at the node h is equivalent to that of $C|_h$ at the initial node, leading us to the next proposition.

Proposition 3. *Let C be a choice correspondence on $\mathcal{G}$. Then, the following three statements are pairwise equivalent for any $h \in H$:*

(a) C is individually rational at a decision node h;

(b) $C|_h$ is individually rational at the initial node;

(c) $C|_h$ satisfies A1, A2, and A3.

Importantly, the choice correspondence $C|_h$ is defined only through the choice correspondence C. Therefore, we can test the individual rationality of players at arbitrary decision nodes on the basis of the same observation C assumed in Section 3.

Remark. Let C be a choice correspondence on $\mathcal{G}$ individually rational at a decision node h. Then, we can uniquely identify the "essential" part of a preference relation of the player at the node h. Note that $C|_h$ is individuallly rational at the initial node (Proposition 3), and $C|_h$ is rationalized by a preference relation on $X|_h$ iff it extends $\succsim^{\emptyset}_{C|_h}$ (Proposition 2). Therefore, where we define a binary relation $\succsim^h_C$ on X by

$$(h, p) \succsim^h_C (h, q) \iff p \succsim^{\emptyset}_{C|_h} q \tag{3}$$

for all $p, q \in X|_h$, it follows that C is individually rational at the decision node h under a preference relation $\succsim$ on X if and only if $\succsim$ extends $\succsim_C^h$. Note that this is a generalization of the same uniqueness result in Proposition 2 relative to an arbitrary decision node h. (In fact, it reduces to Proposition 2 if h is the null history $\emptyset$.)

4.2 Collective rationality

In Section 4.1, we introduced the model of the individual rationality for players at arbitrary decision nodes and verified that we can use the same set of the axioms to test whether the observed choice data admits this model. This means that, given a choice correspondence C on $\mathcal{G}$, we can in principle identify the set of decision nodes at which players make individually rational choice. In this section, we study a special case of such identification, where the choice correspondence C turns out individually rational at *all* decision nodes in the environment. We say that such choice data is *collectively rational* and show that it is in fact closely related to the subgame perfect equilibria, a well-known solution concept for sequential games.

Definition. A choice correspondence C on $\mathcal{G}$ is collectively rational if it is individually rational at a decision node h for every $h \in H$.

Before proceeding with the analysis of the collectively rational choice correspondences in conjunction with the subgame perfect equilibria, we shall clarify testability of a hypothesis that the same players may choose actions more than once in games in $\mathcal{G}$. By definition, for any collectively rational choice correspondence C on $\mathcal{G}$, and for any $h \in H$, there exists a preference relation that rationalizes the observed choice behavior C at the decision node h. But these preference relations in general differ across different decision nodes. The next proposition offers a testable condition written in terms of the revealed preference relations given in (3) that allows us to view the observe choice behavior C as if it is made by the same player at different decision nodes in H.

Proposition 4. *Let H' be a nonempty subset of H, and suppose that C is a choice correspondence on $\mathcal{G}$ individually rational at each decision node in H'. Then, there exists a preference relation $\succsim$ on X such that, for all $h \in H'$, C is individually rational at the decision node h' under $\succsim$, provided that*

$$p \ tran\Big(\bigcup_{h \in H'} \succsim_C^h \Big)\ q \quad \Longrightarrow \quad q \succ_C^h p \ \text{ holds for no } h \in H' \tag{4}$$

for any $p, q \in X$.

Therefore, in words, Proposition 4 shows that C can be rationalized at all decision nodes in H' under the same preference relation $\succsim$, given that condition (4) holds. Now, let $t \geq 1$ and H' be the set of all histories h in H of length $t-1$. It is obvious from (3) that the revealed preference relation $\succsim_C^h$ for each $h \in H'$ compares only paths $p \in X$ with $p^{t-1} = h$. This fact allows us to deduce

$$\operatorname{tran}\Big(\bigcup_{h \in H'} \succsim_C^h\Big) \quad = \quad \bigcup_{h \in H'} \succsim_C^h,$$

which in turn implies condition (4). The next result thus follows as a corollary of Proposition 4. Importantly, it shows that, for any collectively rational choice correspondence, we can assume that the choice of actions at all decision nodes in period t is made by the same player without imposing any additional consistency on the observed choice behavior.

Corollary. *Suppose that C is a collectively rational choice correspondence on $\mathcal{G}$. Then, there exists a sequence of preference relations $(\succsim_1, \succsim_2, \ldots)$ such that, for any $t \geq 1$ and $h \in H$ of length $t-1$, C is individually rational at the decision node h under $\succsim_t$.*

Example 5 (Games with alternate moves)**.** Proposition 4 can be further utilized in other contexts. For example, (rather than we consider an environment where linearly ordered players sequentially make choice of actions) suppose that there are only two players who alternately move in the games in $\mathcal{G}$. If we are interested in, say, verifying the individual rationality of the player who moves in odd turns in this environment, then this can be done by testing the individual rationality of an observed choice correspondence C at each decision node $h \in H_{\text{odd}}$ (by Proposition 3) along with the condition that

$$p \operatorname{tran}\Big(\bigcup_{h \in H_{\text{odd}}} \succsim_C^h\Big)\, q \quad \Longrightarrow \quad q \succ_C^h p \quad \text{holds for no } h \in H_{\text{odd}},$$

where H_{odd} is the set of all decision nodes h in H of length of even numbers.[8] By Proposition 4, this condition guarantees the existence of the same preference relation $\succsim$ on X that rationalizes the observed choice C at every decision node $h \in H_{\text{odd}}$.

Now, in order to study the connection of the collectively rational choice correspondences with the game theoretic solution concept, we will first formulate a choice model of subgame perfect equilibrium paths in the present framework. Let $(\succsim_1, \succsim_2, \ldots)$ be a sequence of preference relations on X. For any $G \in \mathcal{G}$, we say that a mapping $s : H(G) \to H(G)$ is *a strategy profile* in a game G if, for all $h \in H(G)$, there exists an action $a \in A$ with $s(h) = (h, a) \in H(G)$. The set of all strategy profiles in G is denoted by $\Sigma(G)$. For any strategy profile $s \in \Sigma(G)$ and any history $h \in H(G)$, we inductively define $s^0(h) := h$ and $s^k(h) := s(s^{k-1}(h))$ for each $k \geq 1$. Then, we define $s^\infty(h)$ as a sequence of actions in A such that, for any $k \geq 1$, its first k terms coincide with those of $s^k(h)$. Note that $s^\infty(h)$ represents a path of actions followed according to the strategy profile s when the game

[8] Note that the length of the decision nodes (or the histories) for the player who moves in odd turns is even numbers.

is played from the decision node h. Also, observe that $s^\infty(h) \in G$ by the definition of the sequential games. We say that a strategy profile $s \in \Sigma(G)$ is *a subgame perfect equilibrium* (or an SPE for short) given $(\succsim_1, \succsim_2, \dots)$ if $s^\infty(h) \succsim_t s^\infty(h, a)$ for any $t \geq 1$, for any history $h \in H(G)$ of length $t - 1$, and for any $a \in A$ with $(h, a) \in H(G)$. Now, we set

$$\mathrm{SPE}(G|\succsim_1, \succsim_2, \dots) = \{s^\infty(\emptyset) : s \text{ is an SPE in } G \text{ given } (\succsim_1, \succsim_2, \dots)\}$$

for every $G \in \mathcal{G}$. This set consists all subgame perfect equilibrium paths in a game $G \in \mathcal{G}$ given preference relations $(\succsim_1, \succsim_2, \dots)$ of the players.

The collective rationality of the choice behavior introduced in this section closely relates to the choice model of subgame perfect equilibrium paths. In fact, we show that a collectively rational choice correspondence chooses only subgame perfect equilibrium paths in any game $G \in \mathcal{G}$ under the same sequence $(\succsim_1, \succsim_2, \dots)$ of preference relations. Moreover, if a game is "pointwise" finite horizon (in the sense that any path in this game is derived by nontrivial choice of actions by only finitely many players), then the collectively rational choice behavior indeed coincides with the choice of subgame perfect equilibrium paths.

Theorem 5. *If C is a collectively rational choice correspondence on $\mathcal{G}$, then there exists a sequence $(\succsim_1, \succsim_2, \dots)$ of preference relations on X such that $C(G) \subseteq SPE(G|\succsim_1, \succsim_2, \dots)$ for all $G \in \mathcal{G}$, and moreover, $C(G) = SPE(G|\succsim_1, \succsim_2, \dots)$ for all $G \in \mathcal{G}$ such that*

$$p \in G \quad \Rightarrow \quad \exists\, t \geq 1 \text{ s.t. } (q \in G \text{ and } q^t = p^t \text{ imply } q = p). \tag{5}$$

Conversely, if C is a choice correspondence on $\mathcal{G}$ for which there exists a sequence $(\succsim_1, \succsim_2, \dots)$ of preference relations on X such that $C(G) = SPE(G|\succsim_1, \succsim_2, \dots)$ for all $G \in \mathcal{G}$, then C is collectively rational.

In particular, any sequential game in $\mathcal{G}$ satisfies condition (5) if X is finite horizon. (See the remark on games with finite horizon in Section 2.2. Example 3 or Example 4 fit in this case for instance.) This observation leads to the following corollary of Theorem 5, which verifies the equivalence of the collectively rational choice behavior with the choice of subgame perfect equilibrium paths. As we can test the collective rationality of a choice correspondence by Proposition 3, this result provides an axiomatic characterization of the choice of SPE paths in games of finite horizon, parallel to the work by Ray and Zhou [12].

Corollary. *Suppose that there exists $T \geq 1$ such that $p, q \in X$ and $p^T = q^T$ imply $p = q$. Then, a choice correspondence C on $\mathcal{G}$ is collectively rational if and only if there exists a sequence $(\succsim_1, \succsim_2, \dots)$ of preference relations on X such that $C(G) = SPE(G|\succsim_1, \succsim_2, \dots)$ for all $G \in \mathcal{G}$.*

Remark. Recall the model of dynamic Bertrand competition with an entry regulation given in Example 2. In this environment, while X is not finite horizon, any sequential game $G \in \mathcal{G}$ still satisfies

(5). Therefore, it follows from Theorem 5, but not from the corollary, that a choice correspondence C in this environment is collectively rational if, and only if, there exists a sequence $(\succsim_1, \succsim_2, \ldots)$ of preference relations on X such that $C(G) = \mathrm{SPE}(G|\succsim_1, \succsim_2, \ldots)$ for all $G \in \mathcal{G}$.

Theorem 5 suggests that the collectively rational choice behavior may apply a certain selection to the subgame perfect equilibrium paths for games of infinite horizon. The next example provides a concrete instance of a collectively rational choice correspondence C such that $C(G)$ indeed excludes some subgame perfect equilibrium paths in some games $G \in \mathcal{G}$ of infinite horizon.

Example 1 (continued)**.** In the tree cutting problem, we shall this time assume that the tree is best appreciated by all players when it is preserved, but each player would rather cut the tree by herself if anyone else would do so. To be specific, suppose that a preference relation of player t, for every t, satisfies $\mathbf{0} \succ_t \mathbf{e}_t \succ_t \mathbf{e}_s$ for all $s > t$. We define a choice correspondence C on $\mathcal{G}$ by

$$C(G) = \begin{cases} \mathbf{0} & \text{if } \mathbf{0} \in G, \\ \mathbf{e}_{t^*} & \text{otherwise} \end{cases} \qquad \text{for any } G \in \mathcal{G},$$

where t^* is the first t such that $\mathbf{e}_t \in G$. Then, C is collectively rational under the preference relations $(\succsim_1, \succsim_2, \ldots)$, while $\mathbf{e}_{t^*}$ is also an SPE path in some games G in $\mathcal{G}$ with $\mathbf{0} \in G$ (such as $G = X$).[9] Therefore, a collectively rational choice correspondence may exclude some SPE paths.

We argue that the theoretical insight from the example above is beyond of technical interest. The normative appeal of the model of collectively rational choice seems solid, particularly if we accept the concept of subgame perfect equilibria. After all, the individual rationality in the collective choice environment is defined in this paper by decomposing the players behavior in subgame perfect equilibria, and we attempt to reconstruct the same solution concept through the idea of the collective rationality. Nevertheless, when we do so, some subgame perfect equilibrium paths may appear more plausible compared to the other. The observation above offers a certain criterion to refine the subgame perfect equilibria from the perspective of the choice theory.

Appendix

A.1 Proofs

Proof of Theorem 1. Let C be a choice correspondence on $\mathcal{G}$ individually rational at the initial node with a rationalizing preference relation $\succsim$ on X. First, observe that, for any $p, q \in X$ with $p_1 \neq q_1$, the representation

[9] Note that the same conclusion holds for *any* sequence of preference relations that rationalizes C. Indeed, we can show that, if $(\succsim'_1, \succsim'_2, \ldots)$ rationalizes C, then it must satisfy $\mathbf{0} \succ'_t \mathbf{e}_t \succ'_t \mathbf{e}_s$ for all t and s with $s > t$. We also note that the same conclusion of this example can be replicated in any environment as long as it embeds the structure X of the tree cutting problem (such as, for example, those of infinite horizon consumption choice problems).

implies

$$p \in C(\{p,q\}) \quad \Leftrightarrow \quad p \succsim q. \tag{6}$$

To verify A1, take any $G \in \mathcal{G}$ and $p \in G$. Note that, by the representation, for any $a \in H_2(G)$,

$$p \in C(G_{p_1} \cup G_a) \quad \Leftrightarrow \quad p \in C(G_{p_1}) \text{ and there exists } q \in C(G_a) \text{ with } p \succsim q. \tag{7}$$

Hence, we have

$$\begin{aligned} p \in C(G) \quad &\Leftrightarrow \quad p \in C(G_{p_1}) \text{ and for any } a \in H_2(G), \text{ there exists } q \in C(G_a) \text{ with } p \succsim q \\ &\Leftrightarrow \quad p \in C(G_{p_1} \cup G_a) \text{ for any } a \in H_2(G) \\ &\Leftrightarrow \quad p \in C(G_{p_1} \cup G') \text{ for any } G \in \Gamma(G), \end{aligned}$$

where the first equivalence follows from the representation, and the second from (7). So, we obtain A1. Next, to show that C satisfies A2, take any distinct G and G' in $\mathcal{G}$ with $\Gamma(G \cup G') = \{G, G'\}$ and $p \in C(G)$. Then, since there obviously exists a $q \in C(G)$ with $p \succsim q$ (by taking $q := p$), the representation implies

$$p \notin C(G \cup G') \quad \Leftrightarrow \quad q \succ p \text{ for all } q \in C(G').$$

By (6), this equivalence readily implies A2. Lastly, we shall verify A3. Note that, for any $p \in X$ and $G \in \mathcal{G}$ with $\{p\} \in \Gamma(\{p\} \cup G)$, the representation implies

$$p \in C(\{p\} \cup G) \quad \Leftrightarrow \quad \text{for any } a \in H_2(G), \text{ there exists a } p' \in C(G_a) \text{ with } p \succsim p'. \tag{8}$$

Fix any $p, q \in X$ and $G \in \mathcal{G}$ such that $\{p\} \in \Gamma(\{p\} \cup G)$ and $\{q\} \in \Gamma(\{q\} \cup G)$. Also, suppose that $p \in C(\{p, r\})$ and $r \in C(\{r, q\})$ for some $r \in X$ with $p_1 \neq r_1 \neq q_1$. Then, we have $p \succsim r \succsim q$ by (6) and thus $p \succsim q$. Now, if $q \in C(\{q\} \cup G)$, then (8) implies that

$$\text{for any } a \in H_2(G), \text{ there exists a } p' \in C(G_a) \text{ with } q \succsim p',$$

which in turn implies that

$$\text{for any } a \in H_2(G), \text{ there exists a } p' \in C(G_a) \text{ with } p \succsim p'$$

as $p \succsim q$. Applying (8) again to the last condition yields $p \in C(\{p\} \cup G)$, as required.

Conversely, let C be a choice correspondence on $\mathcal{G}$ that satisfies A1, A2, and A3. Define binary relations Δ_X, $\trianglerighteq$, $\trianglerighteq'$ on X by $\Delta_X := \{(p,p) : p \in X\}$ (so Δ_X is the diagonal relation on X); $p \trianglerighteq q$ iff $p_1 \neq q_1$ and $p \in C(\{p,q\})$; and $p \trianglerighteq' q$ iff $p_1 = q_1$ and $p \trianglerighteq r \trianglerighteq q$ for some $r \in X$. As in the main text of the paper, we define $\succsim_C^\emptyset := \Delta_X \cup \trianglerighteq \cup \trianglerighteq'$ with its strict part denoted as $\succ_C^\emptyset$. Then, $\succsim_C^\emptyset$ is obviously reflexive. Moreover, since $p \trianglerighteq q$ and $q \trianglerighteq' p$ hold for no $p, q \in X$, we have $\triangleright \subseteq \succ_C^\emptyset$ and $\triangleright' \subseteq \succ_C^\emptyset$. Also, we have

$$p \trianglerighteq q \trianglerighteq r \trianglerighteq p' \;\text{ and }\; p_1 \neq p_1' \quad \text{imply} \quad p \trianglerighteq p' \tag{9}$$

for any $p, q, r, p' \in X$. (To see this, set $G := \{p'\}$, and observe that $\{p\} \in \Gamma(\{p\} \cup G)$, $\{r\} \in \Gamma(\{r\} \cup G)$, and $r \in C(\{r\} \cup G)$. So, A3 implies that $p \in C(\{p\} \cup G) = C(\{p, p'\})$, that is, $p \trianglerighteq p'$.) Now, we shall show that $\succsim_C^\emptyset$ is transitive. To this end, take any $p, p', p'' \in X$, and suppose that $p \succsim_C^\emptyset p' \succsim_C^\emptyset p''$. If any two of these three paths are identical, then we have $p \succsim_C^\emptyset p''$ at once. So, suppose that p, p', p'' are pairwise distinct. Then, the following cases exhaust all possibilities: (i) $p \trianglerighteq p' \trianglerighteq p''$; (ii) $p \trianglerighteq p' \trianglerighteq' p''$; (iii) $p \trianglerighteq' p' \trianglerighteq p''$; (iv) $p \trianglerighteq' p' \trianglerighteq' p''$. Assume (i) holds. If $p_1 = p''_1$, then $p \trianglerighteq' p''$ and thus $p \succsim_C^\emptyset p''$. So, let $p_1 \neq p''_1$. Then, where $G := \{p''\}$, since $\{p\} \in \Gamma(\{p\} \cup G)$ and $\{p''\} = \Gamma(\{p''\} \cup G) = C(\{p''\} \cup G)$, we have $p \in C(\{p\} \cup G) = C(\{p, p''\})$ by A3. Hence, $p \trianglerighteq p''$ and $p \succsim_C^\emptyset p''$. Alternatively, if (ii) holds, then we have $p \trianglerighteq p' \trianglerighteq r \trianglerighteq p''$ for some $r \in X$ and $p_1 \neq p'_1 = p''_1$, and therefore (9) implies $p \trianglerighteq p''$ and $p \succsim_C^\emptyset p''$ at once. We can similarly show that $p \succsim_C^\emptyset p''$ when (iii) holds. Lastly, suppose that (iv) holds. Then, we have $p \trianglerighteq r \trianglerighteq p' \trianglerighteq r' \trianglerighteq p''$ for some $r, r' \in X$ and $p_1 = p'_1 = p''_1 \neq r'_1$. Then, we have $p \trianglerighteq r'$ by (9), and hence $p \trianglerighteq' p''$ and $p \succsim_C^\emptyset p''$. We have derived $p \succsim_C^\emptyset p''$ in all contingencies, concluding that $\succsim_C^\emptyset$ is transitive. It is well known that any reflexive and transitive binary relation can be extended to a complete and transitive binary relation.[10] Therefore, a preference relation on X that extends $\succsim_C^\emptyset$ exists, and we let $\succsim$ be any such preference relation in what follows. Now, take any $G \in \mathcal{G}$. Then, by the contrapositive of A2, for any $p \in C(G_{p_1})$ and $a \in H_2(G)$ with $a \neq p_1$,

$$p \in C(G_{p_1} \cup G_a) \quad \Leftrightarrow \quad \text{there exists a } q \in C(G_a) \text{ such that } p \in C(\{p, q\})$$

and hence

$$p \in C(G_{p_1} \cup G_a) \quad \Leftrightarrow \quad \text{there exists a } q \in C(G_a) \text{ such that } p \succsim q \tag{10}$$

as $\succsim$ extends $\trianglerighteq$. Also, note that (10) trivially holds for any $p \in C(G_{p_1})$ and $a \in H_2(G)$ with $a = p_1$. In turn, A1 implies that, for any $p \in G$,

$$\begin{aligned} p \in C(G) \quad &\Leftrightarrow \quad \text{for any } G' \in \Gamma(G),\ p \in C(G_{p_1} \cup G') \\ &\Leftrightarrow \quad \text{for any } a \in H_2(G),\ p \in C(G_{p_1} \cup G_a) \\ &\Leftrightarrow \quad p \in C(G_{p_1}),\ \text{and for any } a \in H_2(G),\ p \in C(G_{p_1} \cup G_a) \\ &\Leftrightarrow \quad p \in C(G_{p_1}),\ \text{and for any } a \in H_2(G),\ \text{there exists a } q \in C(G_a) \text{ such that } p \succsim q, \end{aligned}$$

where the last equivalence follows from (10). As G is arbitrary, this verifies that C is individually rational at the initial node under the preference relation $\succsim$. The proof is complete. □

Proof of Proposition 2. For the first half of the proposition, suppose that a choice correspondence C on $\mathcal{G}$ is individually rational at the initial node. Then, C satisfies A1-A3 by Theorem 1. Moreover, in the proof of Theorem 1, assuming that C satisfies A1-A3, we have already verified that $\succsim_C^\emptyset$ is a preorder on X and that any preference relation $\succsim$ that extends $\succsim_C^\emptyset$ rationalizes C at the initial node. So, conversely, suppose that C is individually rational at the initial node under a preference relation $\succsim$ on X. Where $\trianglerighteq$ and $\trianglerighteq'$ are as defined in the proof of Theorem 1, it follows that

$$\begin{aligned} p \trianglerighteq q \quad &\Rightarrow \quad p_1 \neq q_1 \text{ and } p \in C(\{p, q\}) \quad \Rightarrow \quad p \succsim q; \\ p \triangleright q \quad &\Rightarrow \quad p_1 \neq q_1 \text{ and } \{p\} = C(\{p, q\}) \quad \Rightarrow \quad p \succ q. \end{aligned}$$

[10] This result was first proved by Szpilrajn [19].

Hence, $\succsim$ extends $\trianglerighteq$. This in turn implies that

$$\begin{aligned} p \trianglerighteq' q \quad &\Rightarrow \quad p \trianglerighteq r \trianglerighteq q \text{ for some } r \in X \\ &\Rightarrow \quad p \succsim r \succsim q \text{ for some } r \in X \\ &\Rightarrow \quad p \succsim q; \\ p \vartriangleright' q \quad &\Rightarrow \quad p \vartriangleright r \trianglerighteq q \text{ or } p \trianglerighteq r \vartriangleright q \text{ for some } r \in X \\ &\Rightarrow \quad p \succ r \succsim q \text{ or } p \succsim r \succ q \text{ for some } r \in X \\ &\Rightarrow \quad p \succ q, \end{aligned}$$

verifying that $\succsim$ extends $\trianglerighteq'$. This completes to show that $\succsim_C^\emptyset = \Delta_X \cup \trianglerighteq \cup \trianglerighteq' \subseteq \succsim$ and $\succ_C^\emptyset = \vartriangleright \cup \vartriangleright' \subseteq \succ$, that is, $\succsim$ extends $\succsim_C^\emptyset$. For the second half of the proposition, take any preorder $\trianglerighteq$ on X, and suppose that condition (1) holds. To reduce notation, let us write $\trianglerighteq^* = \mathrm{tran}(\succsim_C^\emptyset \cup \trianglerighteq)$. Obviously, we have $\succsim_C^\emptyset \cup \trianglerighteq \subseteq \trianglerighteq^*$. If $p \succ_C^\emptyset q$, then we have $p \trianglerighteq^* q$ (as $\succsim_C^\emptyset \subseteq \trianglerighteq^*$) and not $q \trianglerighteq^* p$ (or otherwise (1) implies not $p \succ_C^\emptyset q$). So, $\trianglerighteq^*$ extends $\succsim_C^\emptyset$. Similarly, we can show that $\trianglerighteq^*$ extends $\trianglerighteq$. Conclusion: $\trianglerighteq^*$ is a preorder on X that extends both $\succsim_C^\emptyset$ and $\trianglerighteq$. Now, by Szpilrajn's theorem (see footonote 10), there exists a preference relation $\succsim$ on X that extends $\trianglerighteq^*$. Since $\succsim$ extends $\succsim_C^\emptyset$, it rationalizes C at the initial node by the first half of this proposition. As $\succsim$ also extends $\trianglerighteq$, the proof is now complete. □

Proof of Proposition 3. (a)⇒(b). Let $h \in H$, and suppose that C is individually rational at a decision node h under a preference relation $\succsim$ on X. Define a binary relation $\succsim'$ on $X|_h$ by $p \succsim' q$ iff $(h, p) \succsim (h, q)$ for any $p, q \in X|_h$. It is straightforward to show that $\succsim'$ is complete and transitive. Moreover, for any $G \in \mathcal{G}|_h$, $p \in C|_h(G)$ iff $(h, p) \in C(\{(h, p') : p' \in G\})$ by (2), which is in turn equivalent to

$$(h, p) \in C(G'_{(h,p_1)}),\ \text{and}\ \forall\, a \text{ with } (h, a) \in H(G'),\ \exists\, (h, q) \in C(G'_{(h,a)}) \text{ with } (h, p) \succsim (h, q) \tag{11}$$

by the representation, where $G' := \{(h, p') : p' \in G\}$. But, observing that (2) implies

$$\begin{aligned} (h, p) \in C(G'_{(h,p_1)}) \quad &\Leftrightarrow \quad (h, p) \in C(\{(h, p') : p' \in G, p'_1 = p_1\}) \\ &\Leftrightarrow \quad p \in C|_h(G_{p_1}); \\ (h, a) \in H(G') \quad &\Leftrightarrow \quad a \in H_2(G); \\ (h, q) \in C(G'_{(h,a)}) \quad &\Leftrightarrow \quad (h, q) \in C(\{(h, p') : p' \in G, p'_1 = a\}) \\ &\Leftrightarrow \quad q \in C|_h(G_a); \end{aligned}$$

the condition (11) is further equivalent to say that

$$p \in C|_h(G_{p_1}),\ \text{and}\ \forall\, a \in H_2(G),\ \exists\, q \in C|_h(G_a) \text{ with } p \succsim' q.$$

As $G \in \mathcal{G}|_h$ is arbitrary, $C|_h$ is hence individually rational at the initial node under $\succsim'$.

(b)⇒(a). Take any $h \in H$, where we let $t \geq 1$ equal the length of the sequence h plus one, and suppose that $C|_h$ is individually rational at the initial node under a preference relation $\succsim$ on $X|_h$. Define a binary relation $\succsim'$ on X by, for any $p, q \in X$, $p \succsim' q$ iff either (i) $p = q$ or (ii) $p = (h, p')$, $q = (h, q')$, and $p' \succsim q'$. It is easy to see that $\succsim'$ is reflexive and transitive. So, there exists a preference relation on X that extends $\succsim'$. (See footnote 10.) Abusing notation, we shall denote this preference relation on X by $\succsim'$. Fix any $G \in \mathcal{G}$

with $h \in H(G)$ and any $p \in G_h$. Let $G' = \{q' \in X|_h : (h, q') \in G_h\}$, and then we have $p = (h, p')$ for some $p' \in G'$. It follows that

$$\begin{aligned} p \in C(G_h) \quad &\Leftrightarrow \quad (h, p') \in C(\{(h, q') : q' \in G'\}) \\ &\Leftrightarrow \quad p' \in C|_h(G') \\ &\Leftrightarrow \quad p' \in C|_h(G'_{p'_1}), \text{ and } \forall\, a \in H_2(G'),\ \exists\, p'' \in C|_h(G'_a) \text{ with } p' \succsim p'', \end{aligned} \tag{12}$$

where the second equivalence follows from (2) and the last from the representation. Since we can show that

$$\begin{aligned} p' \in C|_h(G'_{p'_1}) \quad &\Leftrightarrow \quad (h, p') \in C(\{(h, q') : q' \in G'_{p'_1}\}) \\ &\Leftrightarrow \quad p \in C(G_{(h,p'_1)}) \\ &\Leftrightarrow \quad p \in C(G_{(h,p_{t+1})}); \\ a \in H_2(G') \quad &\Leftrightarrow \quad (h, a) \in H(G); \\ p'' \in C|_h(G'_a) \quad &\Leftrightarrow \quad (h, p'') \in C(\{(h, q') : q' \in G'_a\}) \\ &\Leftrightarrow \quad (h, p'') \in C(G_{(h,a)}); \end{aligned}$$

the condition (12) is further equivalent to say that

$$p \in C(G_{h,p_t}), \text{ and } \forall\, a \text{ with } (h, a) \in H(G),\ \exists\, q \in C(G_{(h,a)}) \text{ with } p \succsim' q.$$

As $G \in \mathcal{G}$ with $h \in H(G)$ is arbitrary, this shows that C is individually rational at the decision node h under the preference relation $\succsim'$. The equivalence (b)$\Leftrightarrow$(c) follows from Theorem 1 at once. □

Proof of Proposition 4. Suppose that condition (4) holds, and we shall write

$$\trianglerighteq^* := \operatorname{tran}\Big(\bigcup_{h \in H'} \succsim_C^h \Big)$$

to reduce notation. Then, obviously, $\succsim_C^h \subseteq \trianglerighteq^*$ for every $h \in H'$. Moreover, for any $h \in H'$, if $p \succ_C^h q$, then we have $p \trianglerighteq^* q$ (as $\succsim_C^h \subseteq \trianglerighteq^*$) and in addition not $q \trianglerighteq^* p$ by the contrapositive of (4). Therefore, $\trianglerighteq^*$ extends $\succsim_C^h$ for all $h \in H'$, Now, by Szpilrajn's theorem (footnote 10), let $\succsim$ be a preference relation on X that extends $\trianglerighteq^*$. Then, for every $h \in H'$, the preference relation $\succsim$ rationalizes C at the decision node h as it extends $\succsim_C^h$. (See the remark below Proposition 3.) The proof is complete. □

Proof of Theorem 5. Let C be a collectively rational choice correspondence on $\mathcal{G}$. As noted in the remark in Section 4.2, then there exists a sequence $(\succsim_1, \succsim_2, \ldots)$ of preference relations on X such that, for any $t \geq 1$ and any $h \in H$ of length $t-1$, C is individually rational at the decision node h under $\succsim_t$. Now, fix an arbitrary $G \in \mathcal{G}$ and $p \in C(G)$. We inductively define a map $\sigma : H(G) \mapsto G$ such that, for any $t \geq 1$ and any $h \in H_t(G)$,

(a) $\sigma(h) \in C(G_h)$,

(b) $\sigma(h, a^*) = \sigma(h)$ if a^* is the tth term of $\sigma(h)$,

(c) $\sigma(h) \succsim_t \sigma(h, a)$ for all $a \in A$ with $(h, a) \in H(G)$.

First, we define σ on $H_1(G)$ by setting $\sigma(\emptyset) = p$. This way of setting σ obviously satisfies the condition (a) above on $H_1(G)$. Then, for any $t \geq 1$, assume that we have defined σ on $H_1(G) \cup \cdots \cup H_t(G)$ in such a way to meet the three conditions above whenever σ is defined. Take any $h \in H_t(G)$. Since $\sigma(h) \in C(G_h)$. and C is individually rational at the decision node h under $\gtrsim_t$, we have

$$\sigma(h) \in C(G_{(h,a^*)}), \text{ and for any } a \text{ with } (h,a) \in H(G), \text{ there exists a } q \in C(G_{(h,a)}) \text{ with } \sigma(h) \gtrsim_t q, \tag{13}$$

where a^* is the tth term of the path $\sigma(h)$. So, we set $\sigma(h, a^*) = \sigma(h) \in C(G_{(h,a^*)})$ and, for any $a \neq a^*$ with $(h, a) \in H(G)$, $\sigma(h, a)$ as a path in $C(G_{(h,a)})$ such that $\sigma(h) \gtrsim_t \sigma(h, a)$, the existence of which is assured by (13). As $h \in H_t(G)$ is arbitrary in this argument, this way we defined σ on $H_{t+1}(G)$, and it follows from the construction that σ satisfies the conditions (a)-(c) on $H_1(G) \cup \cdots \cup H_{t+1}(G)$. Given the map σ on $H(G)$ thus obtained, we next define a strategy profile $s : H(G) \to H(G)$ in the game G by setting $s(h) = (h, a^*)$ where a^* is the tth term of $\sigma(h)$ for any $t \geq 1$ and any $h \in H(G)$ of length $t - 1$. The condition (b) of σ then implies that $s^\infty(h) = \sigma(h)$, and the condition (c) implies that $s^\infty(h) \gtrsim_t s^\infty(h, a)$, for any $t \geq 1$, any $h \in H(G)$ of length $t - 1$, and any $a \in A$ with $(h, a) \in H(G)$. So, s is an SPE profile, and $p = \sigma(\emptyset) = s^\infty(\emptyset) \in \mathrm{SPE}(G|\gtrsim_1, \gtrsim_2, \ldots)$, as claimed.

Next, maintaining the assumption that C is collectively rational under the sequence $(\gtrsim_1, \gtrsim_2, \ldots)$ of preference relations, we take an arbitrary $G \in \mathcal{G}$ that satisfies (5). Observe that, for any $p \in G$, the collective rationality of C implies that

$$p \in C(G_{p^{t-1}}) \quad \Rightarrow \quad p \in C(G_{p^t})$$

for all $t \geq 1$, while (5) implies that $G_{p^t} = \{p\}$ and thus $p \in C(G_{p^t})$ for some $t \geq 1$. Put together, it follows that, for any $p \in G$, there exists $\tau(p) \geq 0$ such that

$$t < \tau(p) \quad \Rightarrow \quad p \notin C(G_{p^t}),$$
$$t \geq \tau(p) \quad \Rightarrow \quad p \in C(G_{p^t}).$$

Now, fix any $p \in \mathrm{SPE}(G|\gtrsim_1, \gtrsim_2, \ldots)$, and suppose that $p \notin C(G)$ towards a contradiction. Then, $\tau(p) \geq 1$. For simplicity, assume that $\tau(p) = 1$, so that $p \notin C(G)$ whereas $p \in C(G_{p_1})$. (The proof works in the same way without this assumption.) Since C is individually rational at the initial node, the fact that p is not chosen in G while it is chosen in G_{p_1} implies that player 1 finds the path suboptimal. To be specific, there exists $a \neq p_1$ with $a \in H(G)$ such that

$$q \in C(G_a) \quad \Rightarrow \quad q \succ_1 p. \tag{14}$$

However, since p is an SPE path in G, there must exist an SPE path $p' \in \mathrm{SPE}(G_a|\gtrsim_1, \gtrsim_2, \ldots)$ in the continuation game G_a such that $p \gtrsim_1 p'$. Then, by (14), $p' \notin C(G_a)$. In summary, we began from the path $p \in \mathrm{SPE}(G|\gtrsim_1, \gtrsim_2, \ldots) \setminus C(G)$ and verified the existence of a path $p' \in \mathrm{SPE}(G_h|\gtrsim_1, \gtrsim_2, \ldots) \setminus C(G_h)$, distinct from p, in a continuation game G_h of G. But then, by repeating the same logic to the path p', we can find a path $p'' \in \mathrm{SPE}(G_{h'}|\gtrsim_1, \gtrsim_2, \ldots) \setminus C(G_{h'})$, distinct from p', in a continuation game G'_h of G_h. By induction, therefore, we can show that there exists a sequence $\psi : \mathbb{N} \to G$ of distinct paths in G such that $\psi(m)^n = \psi(n)^n$ whenever $m > n$.[11] Now, define $q = (\psi(1)_1, \psi(2)_2, \psi(3)_3, \ldots)$. Then, by the closedness property in the defi-

[11] Reminder: we denote, for any path $p \in X$, by p_t the tth term of p and by $p^t = (p_1, \ldots, p_t)$ the first t terms of p. As these notations are reserved, we here use a map $\psi : \mathbb{N} \to G$ to represent a sequence of paths. This sequence is simply constructed by $\psi(1) = p$, $\psi(2) = p'$, $\psi(3) = p''$, and so on, in this proof.

nition of sequential games, we have $q \in G$. Yet, for any $t \geq 1$, $q \neq \psi(t) \in G_{q^t}$, which violates the hypothesis (5). A necessary contradiction is established.

Lastly, suppose that C is a choice correspondence on $\mathcal{G}$ such that $C(G) = \mathrm{SPE}(G|\succsim_1, \succsim_2, \ldots)$ for all $G \in \mathcal{G}$ under a sequence $(\succsim_1, \succsim_2, \ldots)$ of preference relations on X, In this proof, we show that C is individually rational at the initial node. (Given that C coincides with the choice of the SPE paths in all games in $\mathcal{G}$, which includes all continuation games, the same logic works to show that C is individually rational at an arbitrary decision node.) Fix any $G \in \mathcal{G}$, and assume that $p \in C(G)$, that is, the condition (a) in the definition of the individual rationality. Then, there exists a strategy profile s in G such that $p = s^\infty(\emptyset)$ and

$$s^\infty(h) \succsim_t s^\infty(h, a) \tag{15}$$

for any $t \geq 1$, any $h \in H(G)$ of length $t-1$, and any $a \in A$ with $(h, a) \in H(G)$. For any $a \in H_2(G)$, define

$$s_a(h) = \begin{cases} a & \text{if } h = \emptyset, \\ s(h) & \text{otherwise.} \end{cases}$$

(This is a strategy profile where all but player 1 follow s while player 1 plays the action a at the initial node.) It follows from (15) that s_a is an SPE strategy profile in G_a. Moreover, $s_a^\infty(\emptyset) = s^\infty(a)$ for all $a \in H_2(G)$, and, in particular, $s_{p_1}^\infty(\emptyset) = s^\infty(p_1) = s^\infty(\emptyset) = p$. Since s_{p_1} is an SPE in G_{p_1}, we have $p = s_{p_1}^\infty(\emptyset) \in C(G_{p_1})$. Also, for any $a \in H_2(G)$, since s_a is an SPE in G_a, $s^\infty(a) = s_a^\infty(\emptyset) \in C(G_a)$. Then, by applying (15) with $t = 1$ and $h = \emptyset$, we have

$$p = s^\infty(\emptyset) \succsim_1 s^\infty(a)$$

for all $a \in H_2(G)$. This completes to verify the condition (b). Conversely, suppose that $p \in C(G_{p_1})$, and for any $a \in H_2(G)$, there exists a $q \in C(G_a)$ with $p \succsim_1 q$. Then, there exists a strategy profile s_{p_1} in G_{p_1} such that $p = s_{p_1}^\infty(\emptyset)$ and

$$s_{p_1}^\infty(h) \succsim_t s_{p_1}^\infty(h, b) \tag{16}$$

for any $t \geq 1$, any $h \in H(G_{p_1})$ of length $t-1$, and any $b \in A$ with $(h, b) \in H(G_{p_1})$. Also, for each $a \in H_2(G)$ distinct from p_1, there exists a stretegy profile s_a in G_a such that $p \succsim_1 s_a^\infty(\emptyset)$ and

$$s_a^\infty(h) \succsim_t s_a^\infty(h, b) \tag{17}$$

for any $t \geq 1$, any $h \in H(G_a)$ of length $t-1$, and any $b \in A$ with $(h, a) \in H(G_a)$. We define a strategy profile s in G by

$$s(h) = \begin{cases} s_{p_1}(h) & \text{if } h = \emptyset \text{ or } h_1 = p_1 \\ s_{h_1}(h) & \text{otherwise.} \end{cases}$$

Then, $s^\infty(\emptyset) = s_{p_1}^\infty(\emptyset) = p \succsim_1 s_a^\infty(\emptyset) = s^\infty(a)$ for all $a \in H_2(G)$. Also, (16) and (17) imply that

$$s^\infty(h) = s_{h_1}^\infty(h) \succsim_t s_{h_1}^\infty(h, b) = s^\infty(h, b)$$

for any $t \geq 2$, any $h \in H(G)$ of length $t-1$, and any $b \in A$ with $(h, b) \in H(G)$. Hence, s is an SPE in G, and $p = s^\infty(\emptyset) \in C(G)$, as we sought. □

A.2 Independence of the axioms

Only A1, but neither A2 nor A3, has implications on $C(G)$ for games $G \in \mathcal{G}$ with $|H_2(G)| \geq 3$ and $|G_a| \geq 2$ for each $a \in H_2(G)$. So, for example, let $X = \{(a, b, \emptyset, \ldots) : a \in \{1, 2, 3\}, b \in \{1, 2\}\}$, and define the lexicographic relation $\succsim^L$ on X by

$$(a, b, \emptyset, \ldots) \succsim^L (a', b', \emptyset, \ldots) \quad \Longleftrightarrow \quad \text{either} \;\; a > a' \;\; \text{or} \;\; (a = a' \text{ and } b \geq b').$$

Then, a choice correspondence C on $\mathcal{G}$ such that

$$C(G) = \begin{cases} \{(1, 1, \emptyset, \ldots)\} & \text{if } G = X, \\ \{p \in G : p \succsim^L q \text{ for all } q \in G\} & \text{otherwise,} \end{cases}$$

satisfies A2 and A3, but not A1. For A2, let $\mathbf{0} = (0, \emptyset, \ldots)$, $\mathbf{x} = (1, x, \emptyset, \ldots)$, $\mathbf{y} = (1, y, \emptyset, \ldots)$, and $X = \{\mathbf{0}, \mathbf{x}, \mathbf{y}\}$. Define a choice correspondence C on $\mathcal{G}$ by

$$C(\{\mathbf{0}, \mathbf{x}\}) = \{\mathbf{x}\}, \quad C(\{\mathbf{0}, \mathbf{y}\}) = \{\mathbf{0}\}, \quad C(\{\mathbf{x}, \mathbf{y}\}) = \{\mathbf{x}, \mathbf{y}\}, \quad C(\{\mathbf{0}, \mathbf{x}, \mathbf{y}\}) = \{\mathbf{x}\}.$$

Then, C satisfies A1 and A3, but not A2. In particular, it violates A2 since $\mathbf{0} \notin C(\{\mathbf{0}, \mathbf{x}, \mathbf{y}\})$ while $\mathbf{0} \in C(\{\mathbf{0}, \mathbf{y}\})$ and $\mathbf{y} \in C(\{\mathbf{x}, \mathbf{y}\}) = C(\{\mathbf{0}, \mathbf{x}, \mathbf{y}\}_1)$. For A3, let $X = \{\mathbf{x}_0, \mathbf{y}_0, \mathbf{x}_1, \mathbf{y}_1\}$, where $\mathbf{x}_0 = (0, x, \emptyset, \ldots)$, $\mathbf{y}_0 = (0, y, \emptyset, \ldots)$, $\mathbf{x}_1 = (1, x, \emptyset, \ldots)$, and $\mathbf{y}_1 = (1, y, \emptyset, \ldots)$. Define a choice correspondence C on $\mathcal{G}$ by

$$C(\{\mathbf{x}_0, \mathbf{x}_1\}) = \{\mathbf{x}_1\}, \quad C(\{\mathbf{x}_0, \mathbf{y}_1\}) = \{\mathbf{x}_0\}, \quad C(\{\mathbf{y}_0, \mathbf{x}_1\}) = \{\mathbf{y}_0\}, \quad C(\{\mathbf{y}_0, \mathbf{y}_1\}) = \{\mathbf{y}_1\} \tag{18}$$

and

$$C(\{\mathbf{x}_0, \mathbf{y}_0\}) = \{\mathbf{x}_0\}, \quad C(\{\mathbf{x}_1, \mathbf{y}_1\}) = \{\mathbf{x}_1\},$$
$$C(\{\mathbf{x}_0, \mathbf{y}_0, \mathbf{x}_1\}) = \{\mathbf{x}_1\}, \quad C(\{\mathbf{x}_0, \mathbf{y}_0, \mathbf{y}_1\}) = \{\mathbf{x}_0\}, \quad C(\{\mathbf{x}_0, \mathbf{x}_1, \mathbf{y}_1\}) = \{\mathbf{x}_1\}, \quad C(\{\mathbf{y}_0, \mathbf{x}_1, \mathbf{y}_1\}) = \{\mathbf{y}_0\},$$
$$C(\{\mathbf{x}_0, \mathbf{y}_0, \mathbf{x}_1, \mathbf{y}_1\}) = \{\mathbf{x}_1\}.$$

Then, C satisfies A1 and A2, but not A3. In particular, the four conditions in (18) violate A3.

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