INVESTMENTS IN SOCIAL TIES, RISK SHARING AND INEQUALITY

ATTILA AMBRUS† AND MATT ELLIOTT⋆

Abstract. This paper provides a framework to study the formation of risk-sharing networks through costly social investments, in particular the inefficiencies and resulting inequality associated with such processes. First, individuals invest in relationships to form a network. Next, neighboring agents negotiate risk-sharing arrangements. There is never underinvestment, but overinvestment is possible and we find a novel trade-off between efficiency and equality. The most stable efficient network also generates the most inequality. When the income correlation structure is generalized by splitting individuals into groups, such that incomes across groups are less correlated but these relationships are more costly, there can be underinvestment across group but not within group. We find that more central agents have better incentives to form across-group links, reaffirming the efficiency inequality trade-off. In general, endogenous network formation in the risk sharing context tends to result in highly asymmetric networks and stark inequalities in consumption levels.

Date: August 15, 2018.
We acknowledge financial support from National Science Foundation (NSF) grant "The economic benefits of investing into social relationships," SES-1429959. We thank Nageeb Ali, Ben Golub, Matt Jackson, Willemien Kets, Cynthia Kinnan, Rachel Kranton, Peter Landry, Horacio Larreguy, Melanie Morten, Kaivan Munshi, Heinrich Nax and Luigi Pistaferri for helpful comments. Section 6 was developed in conjunction first with Arun Chandrasekhar, and later also Erica Fields and Rohini Pande, to support a potential empirical component of the paper. We owe a substantial intellectual debt to them all, but especially Arun Chandrasekhar. We also thank seminar participants at the Calvo-Armengol prize conference, the Harvard-MIT theory seminar, Oxford, Cambridge, Warwick, Microsoft Research, 2012 ThRed conference in Budapest, Syracuse, Ohio State University, Duke, the Tinbergen Institute and the Santa Fe Institute. An earlier version of this paper was circulated under the title "Social Investments, Informal Risk Sharing, and Inequality."
†Department of Economics, Duke University; email: attila.ambrus@duke.edu.
⋆Faculty of Economics, Cambridge University; email: mle30@cam.ac.uk.

1. Introduction

In the context of missing formal insurance markets and limited access to lending and borrowing, incomes may be smoothed through informal risk-sharing agreements that utilize social connections. A large theoretical and empirical literature studies how well informal arrangements replace the missing markets.[1] However, the existing literature does not investigate a potential downside to these agreements: if people's network position affects the share of surplus generated by risk sharing they appropriate, social investments may be distorted and inequality may endogenously arise.[2]

Our starting premise is that social networks are endogenous and that their structure affects how the surplus from risk sharing is split. There is growing empirical evidence that risk-sharing networks respond to financial incentives, and that in general risk-sharing networks form endogenously, in a way that depends on the economic environment: see for example recent work by Binzel et al. (2017) and Banerjee et al. (2014b,c), which in different contexts look at how social networks respond to the introduction of financial instruments such as savings vehicles or microfinance. Our main goal is to develop a theoretical framework that can be used to think about the endogeneity of risk sharing networks, and to interpret how these networks change after certain economic interventions, or more generally after changes in the economic environment.

In this paper we provide an examination of these issues, by considering a simple two stage model. In the first stage villagers invest in costly bilateral relationships, knowing that in the second stage they will reach informal risk-sharing agreements. These agreements determine how the surplus generated by risk sharing is distributed, and they depend on the endogenous structure of the social network from the first stage. In this way we elucidate new costs associated with informal risk-sharing. Once incomes have been realized, risk sharing typically reduces inequality by smoothing incomes. Nevertheless, asymmetric equilibrium networks generate inequality in expected utilities terms. Agents occupying more advantageous positions in the social network appropriate considerably more of the benefits generated by risk sharing. Indeed, seeking to occupy such positions in the network might lead villagers to spend too much time building social capital. Alternatively, if risk sharing with one neighbor generates positive spillovers for other neighbors, there can be too little investment in forming relationships undermining the effectiveness of informal risk-sharing.

Empirical work suggests that both underinvestment and overinvestment in social capital are possible, in different contexts. Austen-Smith and Fryer (2005) cites numerous references from sociology and anthropology, suggesting that members of poor communities allocate

[1] An incomplete list of papers includes Rosenzweig (1988), Fafchamps (1992), Coate and Ravallion (1993), Townsend (1994), Udry (1994), Ligon, Thomas and Worrall (2002), Fafchamps and Gubert (2007), Bloch, Genicot, and Ray (2008), Angelucci and di Giorgi (2009), Jackson, Rodriguez-Barraquer and Tan (2012), Ambrus, Mobius and Szeidl (2014).

[2] Previous works that do consider the network formation problem include Bramoullé and Kranton (2007a,b) in the theoretical literature and Attanasio et al. (2012) in the experimental literature. For a related paper outside the networks framework, see Glaeser et al. (2002).

inefficiently large amounts of time to activities maintaining social ties, instead of productive activities. In contrast, Feigenberg et al. (2013) find evidence in a microfinance setting that it is relatively easy to experimentally intervene and create social ties among people that yield substantial benefits. One explanation for this finding is that there is underinvestment in social relationships.

It is important to study the above aspects of informal risk sharing, both to put related academic work (which often takes social connections to be exogenously given) into context and to guide policy choices. Consider, for example, microfinance. Two central aims of such interventions are to increase the efficiency of investment decisions by providing better access to capital and to reduce inequality. Clearly the value of microfinance then depends on whether informal risk sharing promotes equality or inequality and whether there is underinvestment, overinvestment or efficient investment in social connections. If there is overinvestment, microfinance has a greater scope for efficiency savings in terms of reducing people's allocation of time into social investments. With underinvestment, however, it has more scope for smoothing incomes. If there is neither under- nor overinvestment, it tells us that informal risk sharing is working relatively well as a second-best solution. Understanding which regime applies can help anticipate policy implications and evaluate welfare impacts of interventions.

For analytical tractability and expositional purposes, in the main text we impose several specific assumptions: agents have CARA utilities, their income realizations are jointly normal, and that surplus is negotiated according to a particular bargaining process, split-the-difference negotiations (Stole and Zwiebel (1996)). In the Supplementary Appendix, Section A we extend our main results to much more general settings, dropping all of the specific assumptions above.

In the first stage of interactions, agents choose with whom to form connections. Link formation is costly, as in Myerson (1991) and Jackson and Wolinsky (1996). In the second stage, pairs of agents who have formed a connection *commit* to a bilateral risk-sharing agreement (transfers contingent on income realizations). We assume that these agreements can be perfectly enforced. We investigate agreements satisfying two simple properties. First we require agreements to be pairwise efficient, in that no pair of directly connected agents leave gains from trade on the table.[3] Second, following Stole and Zwiebel (1996), we require the agreements to be robust to "split-the-difference" renegotiations.[4] We show that this leads to

[3]Although we consider a model in which there is perfect bilateral risk sharing, we could easily extend the model so that some income is perfectly observed, some income is private, and there is perfect risk sharing of observable income and no risk sharing of unobservable income. This would be consistent with the theoretical predictions of Cole and Kocherlakota (2001) and the empirical findings of Kinnan (2011). In the CARA utilities setting, such unobserved income outside the scope of the risk-sharing arrangement does not affect our results.

[4]Stole and Zwiebel (1996) model bargaining between many employees and an employer. This scenario can be represented by a star network with the employer at the center.

the surplus being divided by the Myerson value,[5] a network-specific version of the Shapley value.[6] The transfers required to implement the agreements we identify are particularly simple. Each agent receives an equal share of aggregate realized income (as in Bramoullé and Kranton, 2007a) and on top of that state independent transfers are made.[7]

A key implication of the Myerson value determining the division of surplus is that agents who are more centrally located, in a certain sense, receive a higher share of the surplus. Moreover, in our risk-sharing context it implies that agents receive larger payoffs from providing "bridging links" to otherwise socially distant agents than from providing local connections.[8] Empirical evidence supports this feature of our model—see Goyal and Vega-Redondo (2007), and references therein from the organizational literature: Burt (1992), Podolny and Baron (1997), Ahuja (2000), and Mehra et al. (2001).

In the network formation stage, we study the set of pairwise-stable networks (Jackson and Wolinsky, 1996).[9]

Our analysis considers a community comprising of different groups where all agents within each group are ex-ante identical, and establishing links within groups is cheaper than across groups. We also assume that the income realizations of agents within groups are more positively correlated than across groups. Groups can represent different ethnic groups or castes in a given village, or different villages. We find that there can be overinvestment within groups but not underinvestment, whereas across groups underinvestment is likely to be the main concern.

To see the intuition about overinvestment within groups, we first consider the case of homogeneous agents, that is, when there is only one group. Using the inclusion–exclusion principle from combinatorics,[10] we develop a new metric to describe how far apart two agents located in a network are, which we call the Myerson distance. Using this distance we provide a complete characterization of stable networks. We show that for homogenous agents there can never be underinvestment in social connections, as agents establishing an essential link (connecting two otherwise unconnected components of the network) always receive a benefit exactly equal to the social value of the link. However, overinvestment, in the form of redundant links, is possible, and becomes widespread as the cost of link formation decreases.

[5]For related noncooperative foundations for the Myerson value, see Fontenay and Gans (2014) and Navarro and Perea (2013). Slikker (2007) also provides noncooperative foundations, although the game analyzed is not decentralized: offers are made at the coalitional level.

[6]The Myerson value is also often assumed in social networks contexts on normative grounds, as a fair allocation: see a related discussion on pp. 422–425 of Jackson (2010).

[7]For investigations of the division of surplus in social networks in other contexts, see Calvo-Armengol (2001, 2003), Corominas-Bosch (2004), Manea (2011), Kets et al. (2011) and Elliott and Nava (2016).

[8]More precisely, in Section 4 we introduce the concept of Myerson distance to capture the social distance between agents in the network, and show that a pair of agents' payoffs from forming a relationship are increasing in this measure.

[9]Results from Calvo-Armengol and Ilkilic (2009) imply that under some parameter restrictions—for example when agents are ex ante identical—the set of pairwise-stable outcomes is equivalent to the (in general more restrictive) set of pairwise Nash equilibrium outcomes.

[10]See Chapter 10 in van Lint and Wilson (2001).

Our main finding is that even though agents are ex-ante identical, if stable networks are asymmetric, inequality will result. We identify a novel trade-off between efficiency and inequality. Among all possible efficient network structures, we find that the most stable (in the sense of being stable for the largest set of parameter values) results in the most unequal division of surplus (for any inequality measure in the Atkinson class). Conversely, the least stable efficient network entails the most equal division of surplus among all efficient networks. Although agents are ex-ante identical, efficiency considerations push the structure of social connections towards asymmetric outcomes that elevate certain individuals. Socially central individuals emerge endogenously from risk-sharing considerations alone.

Turning attention to the case of multiple groups, we find that across-group underinvestment becomes an issue when the cost of maintaining links across groups is sufficiently high.[11] The reason is that the agents who establish the first connection across groups receive less than the social surplus generated by the link, providing positive externalities for peers in their groups. To consider which agents are best incentivised to provide across-group links we introduce a new measure of network centrality which we term Myerson centrality. Agents more central in this sense have better incentives to provide across-group links. This provides a second force pushing some agents within a group to be more central than others. For example, with two groups, we show that the most stable efficient network structure involves stars within groups, connected by their centers. This reinforces the trade-off between efficiency and equality in the many-groups context. Our model also predicts that more central agents within groups should play a particularly highlighted role, relative to peripheral agents, in maintaining across-group links when the value of informal risk sharing is smaller, as in this case maintaining such links does not provide enough individual benefits for peripheral individuals.

Among the theoretical studies on social networks and informal risk sharing that are most related to ours include Bramoullé and Kranton (2007a,b), Bloch et al. (2008), Jackson et al. (2012), Billand et al. (2012), Ali and Miller (2013, 2016), and Ambrus et al. (2014). Many of these papers focus on the enforcement issues we abstract from, and investigate how social capital can be used to sustain cooperation for lower discount factors than would otherwise be possible. We take a complementary approach and instead focus on the distribution of surplus and the incentives this creates for social investments. One way of viewing our approach is an assumption on the discount factor in a dynamic version of our model. As long as the discount factor is high enough, our equilibrium agreements satisfy the necessary incentive compatibility constraints to be able to be enforced in equilibrium of the dynamic game.

Among the aforementioned papers, Bramoullé and Kranton (2007a,b) and Billand et al. (2012) investigate costly network formation. Bramoullé and Kranton's (2007a,b) model assumes that the surplus on a connected income component is equally distributed, independently of the network structure. This rules out the possibility of overinvestment or inequality,

[11]While across-group overinvestment remains possible, the main concern when across-group link costs are relatively high is underinvestment.

and leads to different types of stable networks than in our model. Instead of assuming optimal risk-sharing arrangements, Billand et al. (2012) assume an exogenously given social norm, which prescribes that high-income agents transfer a fixed amount of resources to all low-income neighbors. This again leads to very different predictions regarding the types of networks that form in equilibrium.

More generally, network formation problems are important. Establishing and maintaining social connections (relationships) is costly, in terms of time and other resources. However, on top of direct consumption utility, such links can yield many economic benefits. Papers studying formation in different contexts include Jackson and Wolinsky (1996), Bala and Goyal (2000), Kranton and Minehart (2001), Hojman and Szeidl (2008), and Elliott (2015). Although we study a specific network formation problem tailored to risk sharing in villages, the general structure of our problem is relevant to other applications.[12]

The remainder of the paper is organized as follows. Section 2 describes risk sharing on a fixed network. In Section 3 we introduce a game of network formation with costly link formation. We focus on network formation within a single group in Section 4 and then turn to the formation of across-group links in Section 5. We discuss how the model might be taken to data in Section 6. Section 7 concludes.

2. Preliminaries and Risk Sharing on a Fixed Network

To study social investments and the network formation problem, first we need to specify what risk-sharing arrangements take place once the network is formed. Below we introduce an economy in which agents face random income realizations, introduce some basic network terminology, and discuss risk-sharing arrangements for a given network.

2.1. **The socio-economic environment.**

We denote the set of agents in our model by $\mathbf{N}$, and assume that they are partitioned into a set of groups $\mathbf{M}$. We let $G : \mathbf{N} \to \mathbf{M}$ be a function that assigns each agent to a group; i.e., if $G(i) = g$ then agent i is in group g. One interpretation of the group partitioning is that $\mathbf{N}$ represents individuals in a region (such as a district or subdistrict), and groups correspond to different villages in the region. Another possible interpretation is that $\mathbf{N}$ represents individuals in a village, and the groups correspond to different castes.

Agents in $\mathbf{N}$ face uncertain income realizations. For tractability, we assume that incomes are jointly normally distributed, with expected value μ and variance σ^2 for each agent.[13] We assume that the correlation coefficient between the incomes of any two agents within the

[12]For a different and more specific application, suppose researchers can collaborate on a project. Each researcher brings something heterogenous and positive to the value of the collaboration, so that the value of the collaboration is increasing in the set of agents involved. Collaboration is possible only when it takes place among agents who are directly connected to another collaborator and surplus is split according to the Myerson value (as in our work, motivated by robustness to renegotiations). Such a setting fits into our framework.

[13]This specification implies that we cannot impose a lower bound on the set of feasible consumption levels. As we show below, our framework readily generalizes to arbitrary income distributions, but the assumption of normally distributed shocks simplifies the analysis considerably.

same group is ρ_w, while between the incomes of any two agents not in the same group it is $\rho_a < \rho_w$.[14] That is, we assume that incomes are more positively correlated within groups than across groups, so that all else equal, social connections across groups have a higher potential for risk sharing.

Although we introduce the possibility of correlated incomes in a fairly stylized way, our paper is one of the first to permit differently correlated incomes between different pairs of agents. Such correlations are central to the effectiveness of risk-sharing arrangements, as shown below.

We refer to possible realizations of the vector of incomes as *states*, and denote a generic state by ω. We let $y_i(\omega)$ denote the income realization of agent i in state ω.

Agents can redistribute realized incomes; hence their consumption levels can differ from their realized incomes. We assume that all agents have constant absolute risk aversion (CARA) utility functions:

$$v(c_i) = -\frac{1}{\lambda} e^{-\lambda c_i}, \tag{1}$$

where c_i is agent i's consumption and $\lambda > 0$ is the coefficient of absolute risk aversion. The assumption of CARA utilities, together with jointly normally distributed incomes, greatly enhances the tractability of our model: as we show below it leads to a transferable utility environment in which the implemented risk-sharing arrangements are relatively simple. This utility formulation can also be considered a theoretical benchmark case with no income effects. We generalize the theory in the Supplementary Appendix, Section A.

2.2. **Basic network terminology.** Before proceeding, we introduce some standard terminology from network theory. A social network L is an undirected graph, with nodes $\mathbf{N}$ corresponding to the different agents, and links representing social connections. Abusing notation we also let L denote the set of links in the network. We will refer to the agents linked to agent i, $\mathbf{N}(i; L) := \{j : l_{ij} \in L\} \subset \mathbf{N}$, as i's *neighbors*. Where there should be no confusion we abuse notation by writing $\mathbf{N}(i)$ instead of $\mathbf{N}(i; L)$. The *degree centrality* of an agent is simply the number of neighbors she has (i.e., the cardinality of $\mathbf{N}(i; L)$). An agent's neighbors can be partitioned according to the groups they belong to. Let $\mathbf{N}_g(i; L)$ be i's neighbors on network L from group g. A *walk* is a sequence of different agents $\{i, k, k', \ldots, k'', j\}$ such that every pair of adjacent agents in the sequence is linked. A *path* is a walk in which all agents are different. The *path length* of a path is the number of agents in the path.

We will sometimes refer to subsets of agents $\mathbf{S} \subseteq \mathbf{N}$ and denote the subgraphs they generate by $L(\mathbf{S}) := \{l_{ij} \in L : i, j \in \mathbf{S}\}$. A subset of agents $\mathbf{S} \subseteq \mathbf{N}$ is *path-connected* on L if, for each $i \in \mathbf{S}$ and each $j \in \mathbf{S}$, there exists a path connecting i and j. For any network there is a unique partition of $\mathbf{N}$ such that there are no links between agents in different partition elements but all agents within a partition element are path-connected. We refer to these partition elements

[14] It is well-known that for a vector of random variables, not all combinations of correlations are possible. We implicitly assume that our parameters are such that the resulting correlation matrix is positive semidefinite.

as *network components*. A *shortest path* between two path-connected agents i and j is a path connecting i and j with a lower path length than any other. The *diameter* of a network component $C \subset L$ is $d(C)$, the maximum value—taken over all pairs of agents in C—of the length of a shortest path. A network component is a *tree* when there is a unique path between any two agents in the component. A *line network* is the unique (tree) network, up to a relabeling of agents, in which there is a path from one (end) agent to the other (end) agent that passes through all other agents. A *star network* is the unique tree network, up to a relabeling of agents, in which one (center) agent is connected to all other agents.

2.3. **Risk-Sharing Agreements.** We assume that income cannot be directly shared between agents $i, j \in \mathbf{N}$ unless they are connected, i.e., $l_{ij} \in L$. However, we let agents' income realizations be publicly observed within their network component, so agents can make transfer arrangements contingent on it. We consider this environment with perfectly observable incomes within a component as a benchmark model, which is a relatively good description of village societies in which people closely monitor each other. It is also straightforward to extend the model so that some income is publicly observed (and shared) while the remaining income is privately observed (and never shared). Results are very similar for this more general setting.[15]

Formally, a risk-sharing agreement on a network L specifies transfer $t_{ij}(\omega, L) = -t_{ji}(\omega, L)$ between neighboring agents i and j for every possible state ω. Abusing notation where there should be no confusion we sometimes drop the second argument and write $t_{ij}(\omega)$ instead of $t_{ij}(\omega, L)$. The interpretation is that in state ω agent i is supposed to transfer $t_{ij}(\omega)$ units of consumption to agent j if $t_{ij}(\omega) > 0$, and receives this amount from agent j if $t_{ij}(\omega) < 0$. Given a transfer arrangement between neighboring agents, agent i's consumption in state ω is $c_i(\omega) = y_i(\omega) - \sum_{j \in \mathbf{N}(i)} t_{ij}(\omega)$. It is straightforward to show that state-contingent consumption plans $(c_i(\cdot))_{i \in \mathbf{N}}$ are feasible, that is they can be achieved by bilateral transfers between neighboring agents, if and only if for each component C, contained agents S, $\sum_{i \in S} c_i(\omega) = \sum_{i \in S} y_i(\omega)$ for every state ω.

A basic assumption we make in our model is that given all other risk-sharing arrangements, an agreement reached by linked agents i and j must leave no gains from trade on the table. There must be no other agreement that can make both i and j strictly better off holding fixed the agreements of other players. We call such transfers *pairwise efficient*.[16]

By the well-known Borch rule (see Borch (1962), Wilson (1968)) a necessary and sufficient condition for this property is that for all neighboring agents i and j,

[15]Kinnan (2011) finds evidence that hidden income can explain imperfect risk sharing in Thai villages relative to the enforceability and moral hazard problems we are abstracting from. Cole and Kocherlakota (2001) show that when individuals can privately store income, state-contingent transfers are not possible and risk sharing is limited to borrowing and lending.

[16]More formally, transfers $\{t_{ij}(\omega, L)\}_{\omega \in \Omega,\, ij: l_{ij} \in L}$ are pairwise efficient for a network L if there is no pair of agents $ij : l_{ij} \in L$ and no alternative transfers $\{t'_{ij}(\omega, L)\}_{\omega \in \Omega,\, ij: l_{ij} \in L}$ such that $t'_{kl}(\omega, L) = t_{kl}(\omega, L)$ for all $kl \neq ij$ and all $\omega \in \Omega$, that gives both i and j strictly higher expected utility.

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)}\right) \Big/ \left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)}\right) = \left(\frac{\partial v_i(c_i(\omega'))}{\partial c_i(\omega')}\right) \Big/ \left(\frac{\partial v_j(c_j(\omega'))}{\partial c_j(\omega')}\right) \tag{2}$$

for every pair of states ω and ω'. But if this holds for all neighboring agents i and j then the same condition must hold for all pairs of agents on a component of L, independently of whether they are directly or indirectly connected. Hence, pairwise-efficient risk-sharing arrangements are equivalent to Pareto-efficient agreements at the component level. For this reason, below we establish some important properties of Pareto-efficient risk-sharing arrangements on components.

Proposition 1 shows that the CARA utilities framework has the convenient property that expected utilities are transferable, in the sense defined by Bergstrom and Varian (1985). This can be used to show that ex-ante Pareto efficiency is equivalent to minimizing the sum of the variances, and it is achieved by agreements that in every state split the sum of the incomes on each network component equally among the members and then adjust these shares by state-independent transfers. The latter determine the division of the surplus created by the risk sharing agreement. We emphasize that this result does not require any assumption on the distribution of incomes, only that agents have CARA utilities.

Proposition 1. *For CARA utility functions certainty-equivalent units of consumption are transferable across agents, and if $L(\mathbf{S})$ is a network component, the Pareto frontier of ex-ante risk-sharing agreements among agents in $\mathbf{S}$ is represented by a simplex in the space of certainty-equivalent consumption. The ex-ante Pareto-efficient risk-sharing agreements for agents in $\mathbf{S}$ are those that satisfy*

$$\min \sum_{i \in \mathbf{S}} \mathrm{Var}(c_i) \quad \textit{subject to} \quad \sum_{i \in \mathbf{S}} c_i(\omega) = \sum_{i \in \mathbf{S}} y_i(\omega) \quad \textit{for every state } \omega,$$

and they consist of agreements of the form

$$c_i(\omega) = \frac{1}{|\mathbf{S}|} \sum_{k \in \mathbf{S}} y_k(\omega) + \tau_i \quad \textit{for every } i \in \mathbf{S} \textit{ and state } \omega,$$

where $\tau_i \in \mathbb{R}$ is a state-independent transfer made to i and $\displaystyle\sum_{k \in \mathbf{S}} \tau_k = 0$.

The proof of Proposition 1 is in Appendix I. Proposition 1 implies that the total surplus generated by efficient risk-sharing arrangements is an increasing function of the reduction in aggregate consumption variance (the sum of consumption variances). For a general distribution of shocks, this function can be complicated. However, if shocks are jointly normally distributed then $c_i = \frac{1}{|\mathbf{S}|} \sum_{k \in \mathbf{S}} y_k + \tau_i$ is also normally distributed, and $\mathrm{E}(v(c_i)) = \mathrm{E}(c_i) - \frac{\lambda}{2} \mathrm{Var}(c_i)$.[17] Hence in this case the total social surplus generated by efficient risk-sharing agreements is proportional to the aggregate consumption variance reduction. This greatly simplifies the computation of surpluses in the analysis below.

[17] See, for example, Arrow (1965).

We use $TS(L)$ to denote the expected total surplus generated by an ex-ante Pareto-efficient risk-sharing agreement on network L, relative to agents consuming in autarky:

$$TS(L) := CE\Big(\Delta\,\mathrm{Var}(L, \emptyset)\Big), \tag{3}$$

where, for $L' \subset L$, $\Delta\,\mathrm{Var}(L, L')$ is the additional variance reduction obtained by efficient risk-sharing on network L instead of L', and $CE(\cdot)$ denotes the certainty-equivalent value of a variance reduction.

For a network L, consisting of a single component, if all agents are from the same group then as there are CARA utility functions and normally distributed incomes

$$TS(L) = CE\Big(\Delta\,\mathrm{Var}(L, \emptyset)\Big) = \frac{\lambda}{2}\Big(\Delta\,\mathrm{Var}(L, \emptyset)\Big) = \frac{\lambda}{2}(n-1)\sigma^2(1-\rho_w) = (n-1)V, \tag{4}$$

where $V := \frac{\lambda}{2}\sigma^2(1-\rho_w)$.

2.4. **Division of Surplus.** The assumption that neighboring agents make pairwise-efficient risk-sharing agreements pins down agreements up to state-independent transfers between neighboring agents, but does not constrain the latter transfers (hence the division of surplus) in any way. To determine these transfers, we follow the approach in Stole and Zwiebel (1996) and require that agreements are robust to split-the-difference renegotiations. This implies that the transfer is set in a way such that the incremental benefit that the link provides to the two agents is split equally between them. This can be interpreted as a social norm. For a detailed motivation of this assumption, and for noncooperative microfoundations, see Stole and Zwiebel (1996) and Brügemann et al. (2018a).

Splitting the incremental benefits of a risk sharing link equally between two agents requires calculating the expected payoffs i and j would receive if they did not have an agreement. We therefore have to consider what agreements would prevail on the network without l_{ij} to find the risk sharing agreements i and j can reach on L, and so on. This results in a recursive system of conditions.

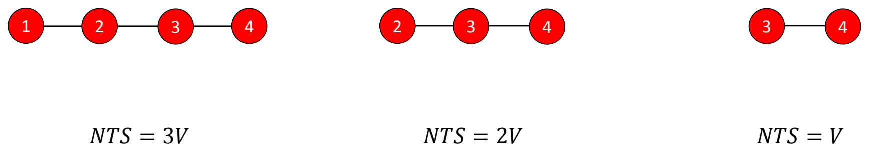
More formally, for a network L a contingent transfer scheme

$$\mathcal{T}(L) := \{t_{ij}(\omega, L')\}_{\omega\in\Omega,\ L'\subseteq L,\ ij:l_{ij}\in L}, \tag{5}$$

specifies all transfers made in all subnetworks of L in all states of the world. The expected utility of agent i on a network $L' \subseteq L$ given a contingent transfer scheme $\mathcal{T}(L)$ is denoted $u_i(L', \mathcal{T}(L))$. Where there should be no confusion, we will abuse notation and drop the second argument.

For any network L, the expected utility vector $(u_1, ..., u_{|\mathbf{N}|})$ is *robust to split-the-difference renegotiation* if there is a contingent transfer scheme $\mathcal{T}(L)$ such that $u_i = u_i(L, \mathcal{T}(L))$ for every $i \in \mathbf{N}$ and the following conditions hold:

(i) $u_i(L') - u_i(L' \setminus \{l_{ij}\}) = u_j(L') - u_j(L' \setminus \{l_{ij}\})$ for every $l_{ij} \in L'$ and $L' \subseteq L$;
(ii) transfers $\{t_{ij}(\omega, L')\}_{\omega\in\Omega,\ ij:l_{ij}\in L'}$ are pairwise efficient for all $L' \subseteq L$.



(A) Line network (B) Three nodes (C) Two nodes

FIGURE 1. To find (gross) expected utilities that are robust to split-the-difference renegotiations on the (formed) line network shown we need to consider the expected utilities that would be obtained on all subnetworks.

Suppose all agents are from the same group, we have CARA utilities, incomes are normally distributed and we want to find payoffs robust to split-the-difference renegotiation for the line network shown in Figure 1a. A first necessary condition is that agents 1 and 2 benefit equally from their link so that $u_1(L) - u_1(L \setminus \{l_{12}\}) = u_2(L) - u_2(L \setminus \{l_{12}\})$. But in order to ensure this condition is satisfied, we need to know $u_1(L \setminus \{l_{12}\})$ and $u_2(L \setminus \{l_{12}\})$. Normalizing the autarky utility of all agents to 0, without the link l_{12} agent 1 is isolated so $u_1(L \setminus \{l_{12}\}) = 0$. However, to find $u_2(L \setminus \{l_{12}\})$ we need to find payoffs for the three node network in Figure 1b. For this network robustness to split-the-difference renegotiation requires that $u_2(L\setminus\{l_{12}\}) - u_2(L\setminus\{l_{12}, l_{23}\}) = u_3(L\setminus\{l_{12}\}) - u_3(L\setminus\{l_{12}, l_{23}\})$. While $u_2(L\setminus\{l_{23}, l_{23}, \}) = 0$, we need to consider the two node network shown in Figure 1c to find $u_3(L \setminus \{l_{12}, l_{23}\})$. For this network, payoffs must satisfy $u_3(L\setminus\{l_{12}, l_{23}\}) - u_3(L\setminus\{l_{12}, l_{23}, l_{34}\}) = u_4(L\setminus\{l_{12}, l_{23}\}) - u_4(L \setminus \{l_{12}, l_{23}, l_{34}\})$. As $u_3(L \setminus \{l_{12}, l_{23}, l_{34}\}) = u_4(L \setminus \{l_{12}, l_{23}, l_{34}\}) = 0$, the above condition simplifies to $u_3(L \setminus \{l_{12}, l_{23}\}) = u_4(L \setminus \{l_{12}, l_{23}\}) = V/2$, where the last equality follows from pairwise efficiency. Considering the three node network again, we now have the condition $u_2(L \setminus \{l_{12}\}) = u_3(L \setminus \{l_{12}\}) - V/2$. As the link l_{23} generates an incremental surplus of V to be split between agents 2 and 3, pairwise efficiency implies that $u_2(L \setminus \{l_{12}\}) = V/2$ and $u_3(L \setminus \{l_{12}\}) = V$. Finally, returning to the line network, we now have $u_1(L) = u_2(L) - V/2$. As the link l_{12} generates incremental surplus of V, $u_1(L) = V/2$ and $u_2(L) = V$.[18]

Below we show that the requirement of robustness to split-the-difference renegotiation implies that the total surplus created by the risk-sharing agreement is divided among agents according to the Myerson value (Myerson 1977, 1980). The Myerson value is a cooperative solution concept defined in transferable utility environments that is a network-specific version of the Shapley value. The basic idea behind it is the same as for the Shapley value.[19] For any order of arrivals of the players, the incremental contribution of an agent i to the total

[18]This argument only outlines why the payoffs $u_1(L) = V/2$ and $u_2(L) = V$ are necessary for robustness to split-the-difference renegotiations. By considering all other subnetworks, it can be shown that the payoffs $u_1(L) = u_4(L) = V/2$ and $u_2(L) = u_3(L) = V$ are the unique payoffs that are robust to split-the-difference renegotiations.

[19]We therefore follow Hart and Moore (1990), among others, in using the Shapley value to study investment decisions.

surplus can be derived as the difference between the total surpluses generated by subgraph $L(\mathbf{S})$ and subgraph $L(\mathbf{S} \setminus \{i\})$ if agents $\mathbf{S} \setminus \{i\}$ arrive before i. It is easy to see that, for any arrival order, the total surplus generated by L gets exactly allocated to the set of all agents. The Myerson value then allocates the average incremental contribution of a player to the total surplus, taken over all possible orders of arrivals (permutations) of the players, as the player's share of the total surplus. Thus, agent i's Myerson value is[20]

$$MV_i(L) := \sum_{\mathbf{S} \subseteq \mathbf{N}} \frac{(|\mathbf{S}|-1)!(|\mathbf{N}|-|\mathbf{S}|)!}{|\mathbf{N}|!} \Big(TS(L(\mathbf{S})) - TS(L(\mathbf{S} \setminus \{i\})) \Big). \tag{6}$$

Proposition 2. *For any network L, any risk-sharing agreement that is robust to split-the-difference renegotiation yields expected payoffs to agents equal to their Myerson values: $u_i(L) = MV_i(L)$.*

Proof. Theorem 1 of Myerson (1980) states that there is a unique rule for allocating surplus for all subnetworks of L that satisfies the requirements of efficiency at the component level (note that this is an implicit requirement in Myerson's definition of an allocation rule) and, what Myerson (1980) defines as the equal-gains principle. Moreover, the expected payoff the above rule allocates to any player i is MV_i. Requirement (i) in our definition of robustness to split-the-difference renegotiation is equivalent to the equal-gains principle as defined in Myerson (1980). Theorem 1 of Wilson (1968) implies that efficiency at the component level is equivalent to pairwise efficiency between neighboring agents, which is requirement (ii) in our definition of robustness to split-the-difference renegotiation. The result then follows immediately from Theorem 1 of Myerson (1980). □

Proposition 2 is a direct implication of Myerson's axiomatization of the value. A special case of Proposition 2 is Theorem 1 of Stole and Zwiebel (1996), which in effect restricts attention to a star network.[21] Our contribution is to point out that their connection between robustness to split-the-difference renegotiations and the Shapley value can be extended to apply to all networks.[22]

The above result shows that any decentralized negotiation procedure between neighboring agents that satisfies two natural properties (not leaving surplus on the table, and robustness

[20]Our assumption that there is perfect risk sharing among path-connected agents ensures that a coalition of path-connected agents generates the same surplus regardless of the exact network structure connecting them. This means that we are in the communication game world originally envisaged by Myerson. We do not require the generalization of the Myerson value to network games proposed in Jackson and Wolinsky (1996), which somewhat confusingly is also commonly referred to as the Myerson value. See Ambrus, Gao and Milan (2016) for a model of informal risk-sharing in which the exact shape of the network matters in terms of the surplus that agents can attain.

[21]Relative to Myerson's axiomatization, Stole and Zwiebel (1996) generate the key system of equations through considering robustness to renegotiations as we describe above, while Myerson wrote down the system of equations based only on fairness considerations. Stole and Zwiebel (1996) also provide non-cooperative bargaining foundations that underpin this system.

[22]Brügemann et al. (2018b) undertake a related exercise regarding the non-cooperative result in Stole and Zwiebel (1996)

to split-the-difference negotiations) leads to the total surplus created by risk-sharing divided according to the Myerson value, and to state-independent transfers between neighboring agents that implement this surplus division. Hence, from now on we assume that in the network formation process, all agents expect the surplus to be divided according to the Myerson value implied by the network that eventually forms.

Although we followed a decentralized approach to get to the implication that surplus is divided by the Myerson value, we note that on normative grounds such a division is also cogent in contexts in which there is a centralized community level negotiation over the division of surplus. This is because the Myerson value is a formal way of defining the fair share of an individual from the social surplus, as his average incremental contribution to the total social surplus (where the average is taken across all possible orders of arrival of different players, in the spirit of the Shapley value).

In the Supplementary Appendix, Section A, we generalize our model by relaxing the CARA utility assumption, relaxing the assumption that incomes are normally distributed and considering a broad class of allocation rules.

3. Investing in Social Relationships

Having defined how formed networks map into risk-sharing arrangements, we can now consider agents' incentives to make investments into social capital, which we think of as the set of relationships that enable risk sharing. We begin by providing the overall framework for the analysis. Then we look at a special case of our model, in which there is a single group. Building on these results we then consider the multiple group case.

In this section we formalize a game of network formation in which establishing links is costly, define efficient networks and identify different types of investment inefficiency.

We consider a two-period model in which in period 1 all agents simultaneously choose which other agents they would like to form links with, and in period 2 agents agree upon the ex-ante Pareto-efficient risk-sharing agreement specified in the previous section (i.e., the total surplus from risk sharing is distributed according to the Myerson value), for the network formed in the first period.[23]

Implicit in our formulation of the timing of the game is the view that relationships are formed over a longer time horizon than that in which agreements are reached about risk sharing. By the time such agreements are being negotiated, the network structure is fixed, and investments into forming social relationships are sunk. In addition, as mentioned in the introduction, the second stage agreements can be viewed as a reduced form treatment of a dynamic game with many state realizations—as long as the discount factor is high enough, our agreements will satisfy the required incentive compatibility constraints for an equilibrium.

[23]For a complementary treatment of network formation when surplus is split according to the Myerson value, see Pin (2011).

In period 1 the solution concept we apply to identify which networks form is pairwise stability. The collection of links formed is social network L, and agent i pays a cost $\kappa_w > 0$ for each link i has to someone in the same group, and $\kappa_a > \kappa_w$ for each links i has to someone from a different group. Normalizing the utility from autarky to 0, we abuse notation[24] and let agent i's *net* expected utility if network L forms be

$$u_i(L) = MV_i(L) - |\mathbf{N}_{G(i)}(i;L)|\kappa_w - \Big(|\mathbf{N}(i;L)| - |\mathbf{N}_{G(i)}(i;L)|\Big)\kappa_a. \tag{7}$$

A network L is *pairwise stable* with respect to expected utilities $\{u_i(L)\}_{i\in\mathbf{N}}$ if and only if for all $i, j \in \mathbf{N}$, (i) if $l_{ij} \in L$ then $u_i(L) - u_i(L \setminus \{l_{ij}\}) \geq 0$ and $u_j(L) - u_j(L \setminus \{l_{ij}\}) \geq 0$; and (ii) if $l_{ij} \notin L$ then $u_i(L \cup l_{ij}) - u_i(L) > 0$ implies $u_j(L \cup l_{ij}) - u_j(L) < 0$. In words, pairwise stability requires that no two players can both strictly benefit by establishing an extra link with each other, and no player can benefit by unilaterally deleting one of his links. From now on we will use the terms pairwise-stable and stable interchangeably.

Existence of a pairwise-stable network in our model follows from a result in Jackson (2003), stating that whenever payoffs in a simultaneous-move network formation game are determined based on the Myerson value, there exists a pairwise-stable network.

Our specification assumes that two agents forming a link have to pay the same cost for establishing the link. However, the set of stable networks would remain unchanged if we allowed the agents to share the total costs of establishing a link arbitrarily.[25] This is because for any link, the Myerson value rewards the two agents establishing the link symmetrically. Hence the agents can find a split of the link-formation cost such that establishing the link is profitable for both of them if and only if it is profitable for both of them to form the link with an equal split of the cost. Given this we stick with the simpler model with exogenously given costs.

A network L is *efficient* when there is no other network L'—and no risk sharing agreement on L'—that can make everyone at least as well off as they were on L and someone strictly better off. Let $|L_w|$ be the number of within-group links, and let $|L_a|$ be the number of across-group links. As expected utility is transferable in certainty-equivalent units, efficient networks must maximize the net total surplus $NTS(L)$:

$$NTS(L) := TS(L) - 2|L_w|\kappa_w - 2|L_a|\kappa_a, \tag{8}$$

Clearly, two necessary conditions for a network to be efficient are that the removal of a set of links does not increase $NTS(L)$ and the addition of a set of links does not increase

[24] In the previous section when investments had already been sunk we used $u_i(L)$ to denote i's expected payoff before link formation costs.

[25] More precisely, we could allow agents to propose a division of the costs of establishing each link as well as indicating who they would like to link to, and a link would then form only if both agents indicate each other and they propose the same split of the cost. A network would then be stable if it is a Nash equilibrium of this expanded network formation game and if there is no new link $l_{ij} \notin L$, and some split of the cost of forming this link, that would make both i and j strictly better off if formed.

$NTS(L)$. If there exists a set of links the removal of which increases $NTS(L)$, we will say there is *overinvestment* inefficiency. If there exists a set of links the addition of which increases $NTS(L)$, we will say there is *underinvestment* inefficiency.[26] A network is *robust to underinvestment* if there is no underinvestment inefficiency and no agent can strictly benefit from deleting a link that would result in underinvestment inefficiency. A network is *robust to overinvestment* if there is no overinvestment inefficiency and no pair of agent i, j can both strictly benefit from creating the link l_{ij}.

We will say that a link l_{ij} is *essential* if after its removal i and j are no longer path-connected while it is *superfluous* if after its removal i and j are still path-connected.

Remark 3. *Preventing overinvestment requires that all links be essential. Superfluous links create no social surplus and are costly. In all efficient networks, therefore, every component must be a tree.*

Real world networks among villagers are a long way from being trees. If our model perfectly captured network formation Remark 3 would imply that there is substantial overinvestment. However, our model is stylized, and this result needs to be applied with caution. For example, while there may be overinvestment, our assumption that all links are costly to form is unlikely to hold. Family ties or the time villagers spend working together might permit relationships to be formed without any additional investment. We discuss in Appendix II how, what we view as the main insights of our results, extend to a setting in which some links are free to form.

In most of the analysis below, we focus on investigating the relationship between stable networks and efficient networks. Additionally, we investigate the amount of inequality prevailing in equilibria in our model. For this, we will use the Atkinson class of inequality measures (Atkinson, 1970). Specifically we consider a welfare function $W : \mathbb{R}^{|\mathbf{N}|} \to \mathbb{R}$ that maps a profile of expected utilities into the real line such that

$$W(u) = \sum_{i \in \mathbf{N}} f(u_i), \tag{9}$$

where $f(\cdot)$ is assumed to be an increasing, strictly concave and differentiable function. The concavity of $f(\cdot)$ captures the social planner's preference for more equal income distributions. Supposing all agents instead received the same expected utility u', we can pose the question what aggregate expected utility is required to keep the level of the welfare function constant.[27] In other words we find the scalar $u' : |\mathbf{N}| f(u') = \sum_{i \in \mathbf{N}} f(u_i)$. Letting $\overline{u} = (1/|\mathbf{N}|) \sum_{i \in \mathbf{N}} u_i$ be the mean expected utility, Atkinson's inequality measure (or index) is given by

[26] Note that these definitions are not mutually exclusive (there can be both underinvestment and overinvestment inefficiency) or collectively exhaustive (inefficient networks can have neither underinvestment nor overinvestment inefficiency if an increase in the net total surplus is only possible by the simultaneous addition and removal of edges).

[27] This exercise is analogous to the certainty equivalent exercise that can be undertaken for an agent facing stochastic consumption.

$$I(f) = 1 - \frac{u'}{\overline{u}} \in [0, 1]. \tag{10}$$

We let $\mathcal{I}$ be the set (class) of Atkinson inequality measures and note that any $I(f) \in \mathcal{I}$ equals zero if and only if all agents receive the same expected utility.[28] Two different inequality measures from the Atkinson class can rank the inequality of two distributions differently. However, certain pairs of distributions are ranked the same way by all members of the class, such as when one distribution is a mean-preserving spread of the other one.

4. Within-Group Network Formation

In this section we assume that $|\mathbf{M}| = 1$, that is, that agents are ex-ante symmetric, and any differences in their outcomes stem from their stable positions on the social network. This will lay the foundations for the more general case considered in the next section.

We begin our investigation by proving a general characterization of the set of stable networks. Recall that a *path* between i and j is a walk in which no agent is visited more than once. If there are K paths between i and j on the network L, we let $\mathbf{P}(i, j, L) = \{P_1(i, j, L), \ldots, P_K(i, j, L)\}$ be the set of these paths. For every $k \in \{1, \ldots, K\}$, let $|P_k(i, j, L)|$ be the cardinality of the set of agents on the path $P_k(i, j, L)$.[29] We can now use these definitions to define a quantity that captures how far away two agents are on a network in terms of the probability that for a random arrival order they will be connected without a direct link when the second of the two agents arrives. We will refer to this distance as the agents' Myerson distance:

$$md(i, j, L) := \frac{1}{2} - \sum_{k=1}^{|\mathbf{P}(i,j,L)|} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le |\mathbf{P}(i,j,L)|} \left(\frac{1}{|P_{i_1} \cup \cdots \cup P_{i_k}|} \right) \right). \tag{11}$$

This expression calculates the probability that for a random arrival order the link l_{ij} will be essential immediately after i arrives,[30] using the classic *inclusion–exclusion principle* from combinatorics. This probability is important because it affects i's incentives to link to j.

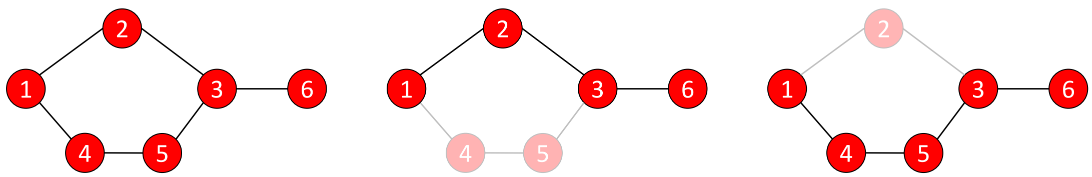
As an illustration, consider the network shown in Panel (A) of Figure 2. The Myerson value allocates each agent their average marginal contribution to total surplus, where the average is taken over all possible arrival orders. For example, for the network shown in Figure 2 consider the arrival order $1, 2, 5, 6, 3, 4$. When agent 1 is added there are no other agents and so no links are formed. Thus 1's marginal contribution to total surplus is 0. Then agent 2 is added and the link l_{12} is formed. This link is essential on this network permitting risk sharing

[28] As $f(\cdot)$ approaches the linear function the social planner cares less about inequality and $I(f) \to 0$. Nevertheless, strict concavity prevents $I(f)$ equaling 0 unless all agents receive the same expected utility.

[29] For example, for a path $P_k(i, j, L) = \{i, i', i'', j\}$, $|P_k(i, j, L)| = 4$ and for a path $P_{k'}(i, j, L) = \{i, i', i''', i'''', j\}$, $|P_{k'}(i, j, L)| = 5$. Finally, we will let $|P_k(i, j, L) \cup P_{k'}(i, j, L)| = 5$ denote number of different agents on path $P_k(i, j, L)$ or path $P_{k'}(i, j, L)$.

[30] If for a given arrival order, agents $\mathbf{S} \subseteq \mathbf{N}$ arrive before i, then l_{ij} is essential immediately after i arrives if it is essential on the network $L(\mathbf{S} \cup \{i\})$.

between agents 1 and 2 that wasn't previously possible. As a result, by equation 4, the total surplus generated by risk sharing increases from 0 to V. Thus 2's marginal contribution to total surplus, for this arrival order, is V. When 5 and 6 are added no new links are formed and no additional risk sharing is possible—their marginal contributions are 0. However, the arrival of 3 results in the formation of the links l_{23}, l_{35} and l_{36}. All of these links are essential and risk sharing among agents $1, 2, 5, 6$ and 3 becomes possible. This increases total surplus to $4V$ by equation 4, so 3's marginal contribution to total surplus is $3V$. Finally, adding 4 the links l_{14} and l_{45} are formed, and this permits risk sharing to also include 4 increasing total surplus to $5V$. So 4's marginal contribution to total surplus is V.



(A) Network (B) Path 1 (C) Path 2

FIGURE 2. Paths connecting nodes 1 and 6.

Whenever a link is formed that is essential for a given arrival order, it contributes V to total surplus, while whenever a link is superfluous for a given arrival order, it makes a marginal contribution of 0 to total surplus.[31] Consider now the incentives agent 1 has to form a superfluous link to agent 6. To calculate this we need to know the probability with which such a link would be essential for a random arrival order. There are three ways in which the link l_{16} might not be essential upon i's arrival. First, with probability $1/2$ agent 6 arrives after agent 1 and the link l_{16} will be formed on 6's arrival instead of 1's. Second, Path 1 shown in Panel (B) of Figure 2 might be present. This will be the case if and only if agents 2, 3 and 6 arrive before agent 1. The probability that agent 1 is last to arrive of these 4 agents is $1/4$. Finally, Path 2 shown in Panel (C) of Figure 2 might be present. This occurs if and only if agents 3, 4 ,5 and 6 arrive before 1. The probability that 1 is last to arrive of these 5 agents is $1/5$.

If these three possibilities were mutually exclusive, then the probability the link l_{16} would be formed and essential upon 1's arrival would be: $1-1/2-1/4-1/5$. The probability that agent 6 arrives after agent 1 is mutually exclusive from the probability that either Path 1 or Path 2 is present, because both these paths need agent 6 to arrive before agent 1. However, it is possible for both Path 1 and Path 2 to be formed upon 1's arrival. Indeed, this occurs if and only if agent 1 is the last agent to arrive, which happens with probability $1/6$. So the probability that at least one of the two paths to agent 6 is present upon 1's arrival is $1/4+1/5-1/6$. We need to subtract the probability $1/6$ to avoid double counting the event

[31]Note that in the arrival order considered in the preceding paragraph, 4's marginal contribution to total surplus would still have been V without the link l_{14} (l_{45}) as long as the link l_{45} (l_{14}) was still formed.

that both paths are present. Thus, the probability that the link l_{16} will be essential upon 1's arrival, is $1 - 1/2 - 1/4 - 1/5 + 1/6 = md(1, 6, L)$.

Lemma 4. *If all agents are from the same group network L is pairwise stable if and only if*

$$\begin{array}{llcll} (i) & md(i,j,L\setminus\{l_{ij}\}) & \geq & \kappa_w/V & \text{for all } l_{ij}\in L, \text{ and} \\ (ii) & md(i,j,L) & \leq & \kappa_w/V & \text{for all } l_{ij}\notin L. \end{array}$$

The proof is relegated to Appendix I. Recall from equation 3 that the social benefits of a link is proportional to the variance reduction it generates. For a single group, if a link l_{ij} is essential in the network $L \cup \{l_{ij}\}$, then this variance reduction is $\Delta\,\mathrm{Var}(L \cup \{l_{ij}\}, L) = (1 - \rho_w)\sigma^2$.

The crucial feature of this expression is that it does not depend on size of the network components the link l_{ij} connects on L. Although in general the size of these components does affect the consumption variance, two effects exactly offset each other.[32] On the one hand, in larger components there are more people to benefit from the essential link. On the other hand, people are already able to smooth there consumption more effectively.

As the social value of a non-essential, or superfluous link, is always zero the total surplus generated by a network L takes a very simple form. Let $\Upsilon(L)$ be the number of network components on L. Then

$$TS(L) = CE\Big(\Delta\,\mathrm{Var}(L,\emptyset)\Big) = \Big(|\mathbf{N}| - \Upsilon(L)\Big)\frac{\lambda}{2}\Big(1-\rho_w\Big)\sigma^2 = \Big(|\mathbf{N}| - \Upsilon(L)\Big)V. \tag{12}$$

Since the surplus created by any essential link is V, the total gross surplus is equal to this constant times the number of network component reductions obtained relative to the empty network.

To consider individual incentives to form links we can use the definition of the Myerson value and consider the average marginal contribution an agent makes to total surplus over all possible arrival orders. Specifically, we want to consider the increase in i's Myerson value due to a link l_{ij}. The link l_{ij} will reduce the number of components in the graph by one when i arrives, relative to the counterfactual component reduction without l_{ij}, if and only if j has already arrived and there is no other path between i and j. In other words, the link increases i's marginal contribution to total surplus if and only if it is essential when i is added. Moreover, for the permutations in which l_{ij} is essential it contributes V to i's marginal contribution to total surplus. Averaging over arrival order, the value to i of the link $l_{ij} \in L$ is $md(i, j, L\setminus\{l_{ij}\})V$, while the value to establishing a new link $l_{ij} \notin L$ is $md(i, j, L)V$.

If a link l_{ij} is essential on L then for any arrival order, there will always be a component reduction of 1 when the later of i or j is added. Therefore, $md(i, j, L) = 1/2$, and l_{ij} will be

[32] Let $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ be the network components of agent i and agent j on network $L \setminus \{l_{ij}\}$, and let $|\mathbf{S}_1| = s_1$ and $|\mathbf{S}_2| = s_2$. Then the sum of consumption variances on $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ (with Pareto efficient risk sharing) are $\frac{s_1+s_1(s_1-1)\rho_w}{s_1}\sigma^2$ and $\frac{s_2+s_2(s_2-1)\rho_w}{s_2}\sigma^2$, respectively. Once S_1 and S_2 are connected through l_{ij}, the sum of consumption variances on $L(\mathbf{S}_1 \cup \mathbf{S}_2)$ becomes $\frac{s_1+s_2+(s_1+s_2)(s_1+s_2-1)\rho_w}{s_1+s_2}\sigma^2$. This implies that the consumption variance reduction induced by the link l_{ij} is $\Delta\,\mathrm{Var}(L \cup \{l_{ij}\}, L) = (1 - \rho_w)\sigma^2$.

formed as long as $V > 2\kappa_w$. As V is the social value of forming the link and $2\kappa_w$ is the total cost of forming it, when all agents are from the same group there is never underinvestment in a stable network or overinvestment in an essential link.

Proposition 5. *If all agents are from the same group then there is never underinvestment in a stable network. Furthermore, there is never overinvestment in an essential link.*

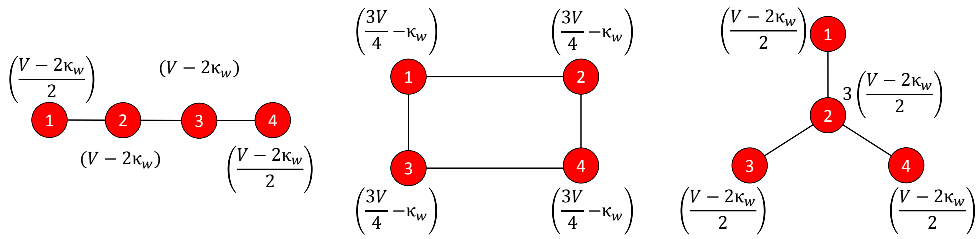
The proof is relegated to Appendix I. When all agents are from the same group Proposition 5 establishes that there is never overinvestment in an essential link, but overinvestment in superfluous links is possible. If the costs of link formation are low enough then agents will receive sufficient benefits from establishing superfluous links to be incentivized to do so. Even if a link l_{ij} is superfluous on L, for some arrival orders it will be essential on the induced subnetwork at the moment when i is added and make a positive marginal contribution to total surplus.[33] An example of such overinvestment is shown in Section B of the Supplementary Appendix.

An immediate implication of Proposition 5 is that if all agents are from the same group and $2\kappa_w > V$ then the only stable network is the empty one and this network is efficient, while if $2\kappa_w < V$ then all stable networks have only one network component (all agents are path-connected). For the remainder of the paper we focus on the parameter range for which the empty network is inefficient for a single group and assume $2\kappa_w < V$. We refer to this as our *regularity condition* and omit it from the statement of subsequent results.

Under this regularity condition the set of efficient networks are the set of tree networks in which all agents are path-connected. In other words, all agents must be in the same component and all links must be essential. We will now focus on which, if any, of these efficient networks are stable. As by Proposition 5 there is never any underinvestment in any stable network the only reason an efficient network will not be stable is if two agents have a profitable deviation by forming an additional (superfluous) link. We therefore focus on investigating what network structures minimize incentives for overinvestment. As we will see, this question is also related to the issue of inequality that different network structures imply.

Figure 3 illustrates three networks: A line (Figure 3a); a circle (Figure 3b) and a star (Figure 3c). While the line and star networks are efficient, the circle network is not as it includes a superfluous link. Among the two efficient networks, the star is more stable than the line. Applying Lemma 4, whenever the line is stable so is the star but there are parameter values for which the star is stable and the line is not. While the star is more stable than the line, it also results in more inequality. The expected utility distribution obtained on the line network can be generated from that obtained on the star network by the best off agent (agent 2) transferring $(V - 2\kappa_w)/2 > 0$ units of expected utility to one of the worst off agents (agent 3). This is enough to ensure that the expected utility distribution on the star is more

[33]Consider, for examples, arrival orders in which i arrives first and j arrives second.



(A) Line (B) Circle (C) Star

FIGURE 3. Three possible network structures for connecting 4 agents and the resulting net payoffs.

unequal than the expected utility distribution on the line for any inequality measure in the Atkinson class. We generalize these insights in Proposition 6.

Proposition 6. *Suppose all agents are from the same group.*

(i) *If there exists an efficient stable network then star networks are stable, and for a non-empty range of parameter specifications only star networks are stable. If a line network is stable then all efficient networks are stable.*

(ii) *For all inequality measures in the Atkinson class, among the set of efficient network, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

The proof is in Appendix I but we provide some intuition after we discuss the result. Proposition 6 states that, in a certain sense, among the set of efficient networks the star is the most stable but maximizes inequality, while the line minimizes inequality but is least stable. This indicates a novel tension between stability/efficiency and inequality. For example, in contrast, Pycia (2012) studies when stable coalitional structures exist and finds that stable coalitions are more likely to exist when the bargaining functions of agents are more equal.

To gain intuition for Proposition 6, recall that Proposition 5 implies that an efficient network will be stable if and only if no pair of players have a profitable deviation in which they form a superfluous link. By Lemma 4 the incentives for two agents to form such a link are strictly increasing in their Myerson distance. Thus, a network is stable if and only if the pair of agents furthest apart from each other, in terms of their Myerson distance, cannot benefit from forming a link. As efficient networks are tree networks, the Myerson distance between any two agents depends only the length of the unique path between them.[34] The longest path between any pair of agents is, by definition, the diameter of the network $d(L)$. So, an efficient network is stable if and only if its diameter is sufficiently small. More precisely,

[34]Suppose d is the number of agents on the unique path connecting i and j. The probability that this path exists when agent i arrives is $1/d$. In addition, if agent j has not yet arrived, which occurs with probability $1/2$, i would not benefit from the link l_{ij}, so i's expected payoff from forming a superfluous link to j is $(1-1/2-1/d)V$. We also note that as d gets large, this converges to $V/2$ which is the value i receives from forming an essential link.

an efficient network L is stable if and only if its diameter is weakly less than $\overline{d}(\kappa_w, V)$, where $\overline{d}(\kappa_w, V)$ is increasing in κ_w, decreasing in V and integer valued.[35]

Let $\mathcal{L}^e(\mathbf{N})$ be the set of efficient networks. Star networks have the smallest diameter among networks within this set, while line networks have the largest diameter among networks within this set. This establishes part (i) of Proposition 6.

To gain intuition for part (ii) a first step is noting that on any efficient network agents' net payoffs are proportional to their degrees (i.e., the number of neighbours they have):[36] $u_i(L) = |\mathbf{N}(i; L)|(V/2 - \kappa_w)$. The key insight is then showing that for any network in the set of efficient networks $\mathcal{L}^e(\mathbf{N})$, the star network can be obtained by rewiring the network (deleting a link $l_{ij} \in L$ and adding a link $l_{ik} \notin L$) in such a way that at each step we increase the degree of the agent who already has the highest degree, reduce the degree of some other agent and obtain a new network in $\mathcal{L}^e(\mathbf{N})$. This process transfers expected utility to the agent with the highest expected payoff from some other agent, thereby increasing inequality for any inequality measure in the Atkinson class. Likewise, we can obtain the line network from any network in the set $\mathcal{L}^e(\mathbf{N})$ by rewiring the network to decrease the degree of the agent with the highest degree at every step. This transfers expected utility from the agent with the highest expected payoff to some other agent, thereby decreasing inequality for any inequality measure in the Atkinson class.

To summarize, this section identifies two novel downsides to informal risk sharing agreements. First, they promote a misallocation of villagers time towards excessive social capital accumulation. Villagers have incentives to form links with a view to becoming more central within the risk sharing network in order to appropriate a larger share of the surplus generated by risk sharing. Secondly, even when investments into social capital are efficient the networks that can be supported in equilibrium generate social inequality, and this translates into (potentially severe) financial inequality.

5. Connections Across Groups

We now generalize our model by permitting multiple groups. These different groups might correspond to people from different villages, different occupations, or different social status groups, such as castes. We will first show that (under our regularity condition) there is still never any underinvestment within a group. However, this does not apply to links that bridge groups. As, by assumption, incomes are more correlated within a group than across a group, there can be significant benefits from establishing such links and not all these benefits accrue to the agents forming the link. Intuitively, an agent establishing a bridging link to another group provides other members of his group with access to a less correlated income stream, which benefits them. As agents providing such bridging links are unable to appropriate all

[35]We show in the proof of Proposition 6 that $\overline{d}(\kappa_w, V) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.
[36]This is also known as an agent's degree centrality.

the benefits these links generate, and these links are relatively costly to establish, there can be underinvestment.

To analyze the incentives to form links within a group, we first need to consider the variance reduction obtained by a within-group link. Such a link may now connect two otherwise separate components consisting of arbitrary distributions of agents from different groups. Suppose the agents in $\mathbf{S}_0 \cup \cdots \cup \mathbf{S}_k$ and the agents in $\widehat{\mathbf{S}}_0 \cup \cdots \cup \widehat{\mathbf{S}}_k$ form two distinct network components, where for every $i \in \{0, ..., k\}$, the agents in $\mathbf{S}_i$ and those in $\widehat{\mathbf{S}}_i$ are all from group i. Consider now a potential link l_{ij} connecting the two otherwise disconnected components. Letting s_0 be the number of agents in group 0, the variance reduction obtained is:[37]

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[(1-\rho_w) + \frac{\sum_{i=0}^{k} \left(\hat{s}_i \sum_{j=0}^{k} s_j - s_i \sum_{j=0}^{k} \hat{s}_j \right)^2}{\left(\sum_{i=0}^{k} s_i\right)\left(\sum_{i=0}^{k} \hat{s}_i\right)\left(\sum_{i=0}^{k} s_i + \hat{s}_i\right)} (\rho_w - \rho_a) \right] \sigma^2. \tag{13}$$

The key feature of this variance reduction is that it is always weakly greater than $(1-\rho_w)\sigma^2$, which is the variance reduction we found in the previous section when all agents were from the same group. Thus, the presence of across-group links only increases the incentives for within-group links to be formed. A within-group link can now give (indirect) access to less correlated incomes from other groups and so is weakly more valuable. This implies that there will still be no underinvestment under our regularity condition that $2\kappa_w < V$.[38] The above reasoning is formalized by Proposition 7.

Proposition 7. *There is no underinvestment between any two agents from the same group in any stable network.*

The proof of Proposition 7 is in Appendix I. While underinvestment is not possible within group, it is possible across groups. An example of this is shown in Section B of the Supplementary Appendix. Although when all agents are from the same group the value of an essential link does not depend on the sizes of the components it connects, the value of an essential link connecting two different groups of agents increases in the sizes of the components. To demonstrate this formally, consider an isolated group that has no across-group connections and consider the incentives for a first such connection to be formed. Thus the first component consist of agents from a single group, say group 0. We let the second component consist of agents from one or more of the other groups (1 to k). The variance reduction obtained by connecting these two components is

[37]By definition

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \operatorname{Var}(L(\mathbf{S}_0, ..., \mathbf{S}_k)) + \operatorname{Var}(L(\widehat{\mathbf{S}_0}, ..., \widehat{\mathbf{S}_k})) - \operatorname{Var}(L(\mathbf{S}_0 \cup \widehat{\mathbf{S}_0}, ..., \mathbf{S} \cup \widehat{\mathbf{S}_k})).$$

Recalling that

$$\operatorname{Var}(L(\mathbf{S}_0, \mathbf{S}_1, ..., \mathbf{S}_k)) = \left(\sum_{i=0}^{k} (s_i + s_i(s_i - 1)\rho_w) + 2\rho_a \sum_{i=0}^{k-1} (s_i \sum_{j=i+1}^{k} s_j) \right) \sigma^2 \Big/ \sum_{i=0}^{k} s_i \ ,$$

some algebra yields the result.

[38]Recall that this regularity condition just requires that it is efficient for two agents in the same group, both without any other connections, to form a link.

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[(1-\rho_w) + \frac{\hat{s}_0 \left(\left(\sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2 \right)}{\left(\sum_{i=1}^k s_i \right) \left(\hat{s}_0 + \sum_{i=1}^k s_i \right)} (\rho_w - \rho_a) \right] \sigma^2, \tag{14}$$

which is increasing in $\hat{s}_0$:

$$\frac{\partial \Delta \operatorname{Var}(L \cup l_{ij}, L)}{\partial \hat{s}_0} = \frac{\left(\sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2}{\left(\hat{s}_0 + \sum_{i=1}^k s_i \right)^2} (\rho_w - \rho_a) \sigma^2 > 0. \tag{15}$$

The inequality follows since $\rho_w > \rho_a$. Thus if agents i and j who connect two otherwise unconnected groups they receive a strictly smaller combined private benefit than the social value of the link. To see why, suppose that on the network L the link l_{ij} is essential, and without l_{ij} there would be two components, the first connecting agents from group $G(i)$ and the second connecting agents from group $G(j) \neq G(i)$. Consider the Myerson value calculation. For arrival orders in which i or j is last to arrive, the value of the additional variance reduction due to l_{ij} obtained upon the arrival of the later of i or j, is the same as its marginal social value, i.e., the value of variance reduction obtained by l_{ij} on L. For any other arrival order the value of variance reduction due to l_{ij} when the later of i or j arrives is strictly less. Averaging over these arrival orders, the link l_{ij} contributes less to i and j's combined Myerson values than its social value, leading to the possibility of underinvestment.

Besides underinvestment, overinvestment is also possible across groups. Forming superfluous links will increase an agent's share of surplus without improving overall risk sharing and can therefore create incentives to overinvest. Nevertheless, when κ_a is relatively high, underinvestment rather than overinvestment in across-group links will be the main efficiency concern. In many settings, within-group links are relatively cheap to establish in comparison to across-group links. For example, when the different groups correspond to different castes, it can be quite costly to be seen interacting with members of the other caste (e.g., Srinivas (1962), Banerjee et al. (2013b)). Motivated by this, and because across-group links are typically far sparser than within-group links, we focus our attention on this parameter region. More concretely, below we investigate what within-group network structures create the best incentives to form across-group links and what network structures minimize the incentives for overinvestment within group. Remarkably, we find that these two forces push within-group network structures in the same direction, and in both cases towards inequality in the society.

We begin by considering within-group overinvestment, which corresponds to the formation of superfluous links within a group. We found in the previous section that when all agents are from the same group the star is the efficient network that minimized the incentives for overinvestment. However, once we include links to other groups, the analysis is more complicated. The variance reduction a within-group link generates is still 0 if the link is superfluous, but when the link is essential it depends on the distribution of agents across the different groups the link grants access to. Moreover, the variance reduction may be decreasing or increasing in the numbers of people in those groups.[39] This makes the Myerson value calculation substantially more complicated. When all agents were from the same group all that mattered was whether the link was essential when added. Now, for each arrival order in which the link is essential, we also need to keep track of the distribution of agents across the different groups that are being connected. Nevertheless, our earlier result generalizes to this setting, although the argument establishing the result is more subtle.

To state the result, it is helpful to define a new network structure. A *center-connected star* network is a network in which all within-group network structures are stars and all across-group links are held by the center agents in these stars. We denote the set of center-connected star networks by $\mathcal{L}^{CCS}$.

Proposition 8. *If any efficient network L is robust to overinvestment within group, then any center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to overinvestment within group. Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to overinvestment within group but L is not.*

The proof of Proposition 8 is in Appendix I. In Proposition 6 we found that when all agents are from the same group, incentives for overinvestment (within group) are minimized by forming a (within-group) star. However, the incentives to form superfluous within-group links are weakly greater when someone within the group holds an across-group link (see equation 13). We can therefore think of the incentives for over-investment we found in Proposition 6 as a lower bound on the minimal incentives we can hope to obtain once there are across-group links. A key step in the proof of Proposition 8 shows that this lower bound is obtained by all center-connected star networks.

Consider a center-connected star network L'. As the agent at the center of a within-group star, agent k, has a link to all agents within the same group, we can focus on the incentives of two non-center agents from the same group, i and j, to form a superfluous link. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. On the induced subnetwork $L'(\mathbf{S})$ either l_{ij} is superfluous or else $k \notin \mathbf{S}$. This implies that no across-group links are present whenever the additional link l_{ij} makes a positive marginal contribution. Hence considering different arrival orders, the average marginal contribution of such a link when it is added is the same on the star network with no across-group links as for a center-connected star network: The lower bound on within-group overinvestment incentives is obtained.

We now consider the within-group network structures that maximize the incentives for an across-group link to be formed. We have already established that the marginal contribution of a first bridging link to the total surplus is increasing in the sizes of the groups it connects. By the Myerson calculation, the agents with the strongest incentives to form such links are

[39]In the case of an essential across-group link that connects agents from just one group to agents from other groups, the comparative statics are unambiguous. In this case, the variance reduction is increasing in the sizes of the groups connected (see inequality (15)).

then those who will be linked to the greatest number of other agents within their group when they arrive. The result below formalizes this intuition.

Let $\mathcal{A}(\mathbf{S}_k)$ be the set of possible arrival orders for the agents in $\mathbf{S}_k$. For any arrival order $A \in \mathcal{A}(\mathbf{S})$, let $\mathbf{T}_i(A)$ be the set of agents to whom i is path-connected on $L(\mathbf{S}')$, where $\mathbf{S}'$ is the set of agents (including i) that arrive weakly before i. Let $T_i^{(m)}$ be a random variable, taking values equal to the cardinality of $\mathbf{T}_i(A)$, where A is selected uniformly at random from those arrival orders in which i is the m-th agent to arrive.

We will say that agent $i \in \mathbf{S}_k$ is more *Myerson central* (from now on, simply more central, for brevity) within his group than agent $j \in \mathbf{S}_k$ if $T_i^{(m)}$ first-order stochastically dominates $T_j^{(m)}$ for all $m \in \{1, 2, ..., |\mathbf{S}_k|\}$.[40] In other words, considering all the arrival orders in which i is the m-th agent to arrive, and all the arrival orders in which j is the m-th agent to arrive, the size of i's component at i's arrival is larger than that of j's at j's arrival in the sense of first-order stochastic dominance.[41] This measure of centrality provides a partial ordering of agents.

Lemma 9. *Suppose agents in $\mathbf{S}_0$ form a network component, and all other agents in $\mathbf{N}$ form another network component. Let $i, i' \in \mathbf{S}_0$ and let $j \notin \mathbf{S}_0$. If i is more central within group than i', then i receives a higher payoff from forming l_{ij} than i' receives from forming $l_{i'j}$:*

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L).$$

The proof is relegated to Appendix I. The key step in the proof pairs the arrival orders of a more central agents with a less central agent, so that in each case the more central agent is connected to weakly more people in the same group upon his arrival, and to the same set of people from other groups. Such a pairing of arrival orders is possible from the definition of centrality, and in particular the first-order stochastic dominance it requires.

Lemma 9 shows that more central agents have better incentives to form intergroup links. We can then consider the problem of maximizing the incentives to form intergroup links by choosing the within-group network structures (networks containing only within-group links). We will say that the within-group network structures that achieve these maximum possible incentives are most robust to underinvestment inefficiency across groups.

Proposition 10. *If any efficient network L is robust to underinvestment across group, then some center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to underinvestment across group. Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications the center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but L is not.*

[40] We also use this notion of centrality to compare the within-group centrality of the same agent on two different network structures. To avoid repetition we do not state the slightly different definition that would apply this situation.

[41] An alternative and equivalent definition is that i is more central than j if there exists a bijection $B : \mathcal{A}(\mathbf{S}_k) \to \mathcal{A}(\mathbf{S}_k)$ such that $|\mathbf{T}_i(A)| \geq |\mathbf{T}_j(B(A))|$ and $A(i) = A'(j)$, where $A(i)$ is i's position in the arrival order A and $A' = B(P)$.

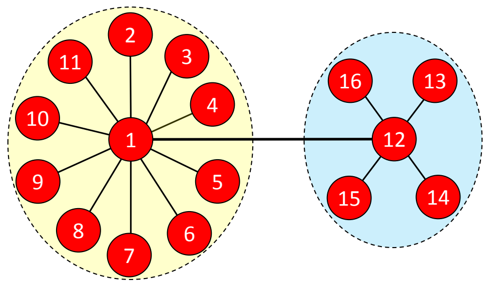


FIGURE 4. Center-connected within-group stars, in a context with two groups.

The proof of Proposition 10 is in Appendix I. Intuition can be gained from Lemma 9. This Lemma shows that agents have better incentives to provide a bridging link across group when they are more central within their own group. Thus to maximize the incentives of an agent to provide an across-group link, we need to maximize the centrality of this agent within group. This is achieved by any network that directly connects this agent to all others in the same group. However, only one of these within-group network structures can be part of an efficient network, and this is the star network, with the agent providing the across-group link at the center.

Figure 4 shows a center-connected star network when there are two groups. As long as it is efficient for these groups to be connected, center-connected star networks and only the center-connected star networks minimize the incentives for within-group overinvestment (by Proposition 9) and minimize the inventives for across-group underinvestment (by Proposition 10).

The above results further reinforce the tension between efficiency and equality. However, one subtlety relative to the one group case is that while the center-connected star network maximizes social inequality among all efficient networks with respect to degree distribution, additional assumptions are required on the parameters of the model to ensure that such networks maximize income inequality.

6. Testable Predictions

We now turn our attention to how our model might be taken to data. Arguably the main economic insights of the paper are through the connection between informal risk sharing and social and economic inequality. With ideal data, these predictions can be tested directly. However, this requires having income data, consumption data and data detailing risk-sharing connections all across many networks that are independent from one another. In principle, this can be done. Viewing each village as a group and collecting information about risk sharing within village and across village within a confined region constitutes an observation, and considering many such regions can generate a dataset with sufficient observations for inference. In this section, we consider two question. First, what might be done with ideal data to test the model, and second what can be done with existing data, referencing a recent paper by Field and Pande (2018).

6.1. **Idealized Empirical Setting.** We focus on the part of the parameter range for our model in which across-group links are very expensive to form relative to within-group links, implying that under-investment rather than over-investment across groups is source of potential inefficiency regarding the formation of across-group risk-sharing relationships. An example might be that marriages across villages are necessary for effective risk-sharing, while within village less intense social relationships are sufficient (Rosenzweig and Stark, 1989). The theory then predicts systematic differences in economic and social inequality, depending on the environment. These differences come from two difference forces that both push in the same direction. First, Lemma 9 shows that more Myerson central agents have better incentives to provide an across-group link. Thus, when across-group links are more valuable, the incentives for very Myerson central individuals to emerge is stronger. Furthermore, from the variance reduction given in equation 13, it is straightforward to show that for an essential across-group bridging link l_{ij}:

$$\begin{aligned}
\frac{\partial \Delta \operatorname{Var}(L, L \cup \{l_{ij}\})}{\partial \sigma^2} &> 0, \\
\frac{\partial \Delta \operatorname{Var}(L, L \cup \{l_{ij}\})}{\partial \rho_a} &< 0.
\end{aligned}$$

Thus when the variance is higher and the across-group income correlation is lower, the benefits from investing in a first across group link are stronger and the incentives to form less equal societies are greater.[42]

A second force that pushes towards more social and economic inequality, is agents' incentives to overinvest in within-group links. More social inequality in terms of agents' Myerson centralities helps lower Myerson distances between agents, reducing their incentives to over-invest in local social capital. The incentives to over-invest are again increasing in σ^2, and decreasing in ρ_a.[43] Predictions P1 and P2 below then follow immediately:

- P1. There is more social and economic inequality in regions with higher σ^2.
- P2. There is more social and economic inequality in regions with lower ρ_a.

We now elaborate on what would constitute an ideal setting for testing these predictions. Suppose that there are many rural villages located in many regions, and that the main economic activity in these villages is farming crops. Further, let risk-sharing be concentrated within village, with some limited across-village risk-sharing within a region and no risk-sharing outside across regions. Each region can be treated as a separate observation of a risk-sharing network structure. Suppose further that all villages within a region farm

[42]The effect of changing ρ_w is ambiguous.

[43]The fact they are decreasing in ρ_a follows from the proof of Proposition 7. They are also decreasing in ρ_w, but as the incentives to form across-group links is ambiguous in ρ_w the overall effect of changing ρ_w is ambiguous.

the same crop,[44] but that these crops are sensitive to rainfall and that the variance in the amount of rainfall per growing season varies systematically across regions. The difference in rainfall variation and the crops farmed across regions will generate exogenous variation in σ^2. Suppose too that there is systematic variation in how susceptible regions are to the transmission of pests and disease across the villages within them, and whether there are different microclimates within the region. This generates exogenous variation in ρ_a across regions. Finally, in this scenario, suppose there is data available over time on the informal risk-sharing network, villagers' incomes before risk-sharing and villagers' incomes after risk sharing or else their consumption.

Of course, there will still be identification concerns to be overcome. For example, a plausible alternative explanation for prediction P1 is that risk-sharing is far from perfect and thus that villages with more variable initial incomes before any risk-sharing takes place will also have less equal incomes after risk-sharing. In cross-sectional data, separating the effects present in our model from this explanation will be hard. However, having data available over time it might be possible to decompose the impact of higher income variance on income inequality into a component that can be attributed to imperfect risk-sharing, and a component that can be attributed to unequal risk-sharing arrangements. The key test of the model would then be whether this second component is statistically and economically significant.

Despite some challenges remaining, the empirical setting we have describe provides observations at the network level and permits the main economic predictions of the theory regarding inequality and informal risk-sharing arrangements to be directly tested. Although at present no such dataset exists to the best of our knowledge, the data available is increasing rapidly and it is possible such data will become available in the near future.

6.2. **Existing Empirical Support.** While the ideal dataset is not currently available, Field and Pande (2018) have nevertheless succeeded in testing some aspects of the theory. They study a setting in which groups correspond to villages, and there is a fair amount of risk-sharing across villages. The researchers conduct a randomized controlled trial in which they pair with a non-governmental organization to introduce microfinance to rural Indian villages. The data includes 185 villages in Tamil Nadu, India and the randomized introduction of microfinance across these villages creates exogenous variation in the value of information risk-sharing. The data does not include information on income, consumption or whom risk-sharing relationships outside of a given village are with, making it hard to test P1 and P2 directly. Instead the authors test a closely related empirical prediction: Villagers with lower Myerson centrality have worse incentives to form across-village risk-sharing links. Further, assuming that the introduction of microfinance into a village reduces the value of an out-of-village risk sharing link, there will be fewer villagers with sufficient incentives to form an

[44]If a mix of crops were farmed within a region this would create potentially useful variation in incomes, but also raise an endogeneity issue as the choices of which crops to farm could be influenced by the risk-sharing arrangements.

out-of-village link in villages where microfinance has been introduced. Thus the association between Myerson centrality and maintaining across-group connections should be higher in treated villages.[45]

P3. In villages where microfinance has been introduced, there will be a stronger association between those villagers that have an out-of-village risk-sharing relationship and their within village Myerson centrality.

Undertaking a regression analysis, with several controls, Field and Pande (2018) find support for Prediction P3 in their data. The relationship between Myerson centrality and out-of-village links is significantly more positive in villages that were randomly chosen to receive formal banking services. As the introduction of microfinance provides truly exogenous variation, their results provide empirical support for a reduction in the value of outside links *causing* there to be a stronger association between agents' Myerson centralities and their propensity to have out-of-village risk-sharing links.

7. Conclusion

Our paper provides a relatively tractable model of network formation and surplus division in a context of risk sharing that allows for heterogeneity in correlations between the incomes of pairs of agents. Such correlations have a sizeable impact on the potential of informal risk sharing to smooth incomes. We investigate the incentives for relationships that enable risk sharing to be formed both within a group (caste or village) and across groups, giving access to less correlated income streams. We find that overinvestment into social relations is likely within a group, but there is potential underinvestment into more costly social connections that bridge different groups. We also find a novel trade-off between equality and efficiency. Thus we identify new downsides to informal risk sharing arrangements that can have important policy implications.

Although we focus our analysis on risk sharing, our conclusions regarding network formation could apply in other social contexts too, as long as the economic benefits created by the social network are distributed similarly to the way they are in our model—a question that requires further empirical investigation. Within the context of risk sharing, a natural next step would be to provide a dynamic extension of the analysis that allows for autocorrelation between income realizations.

[45]Other predictions of the model are less amenable to testing. Consider for example the following four predictions: (1) there are network asymmetries among otherwise observationally similar agents, (2) there is no underinvestment within groups, (3) agents cannot be too far away from each other (in terms of the Myerson distance), (4) agents that have across-group links should be more Myerson central within group. Predictions (2) and (3) depend on an unobservable linking cost parameter and cost heterogeneity would undermine them, while there are many alternative stories, including ones not directly connected to risk sharing, that generate similar predictions. For example, if individuals have heterogenous time budgets that are unobserved, and made random links within and across groups, predictions (1) and (4) are mechanically generated.

References

Ahuja, G. (2000): "Collaboration networks, structural holes, and innovation: A longitudinal study," Administrative science quarterly, 45, 425–455.

Algaba, E., JM Bilbao, J. Fernández, N. Jiménez and J. López (2007): "Algorithms for computing the Myerson value by dividends," Discrete Mathematics Research Progress, 1-13.

Ali, N. and D. Miller (2013): "Enforcing cooperation in networked societies," mimeo UC San Diego.

Ali, N. and D. Miller (2016): "Ostracism and forgiveness,"The American Economic Review, 106.8 (2016): 2329-2348.

Ambrus, A., W. Y. Gao and P. Milan (2016): "Informal Risk Sharing with Local Information," Mimeo.

Ambrus, A., M. Mobius, and A. Szeidl (2014): "Consumption risk-sharing in social networks," American Economic Review, 104, 149-182.

Angelucci, M. and G. De Giorgi (2009): "Indirect effects of an aid program: How do cash injections affect non-eligibles' consumption?," American Economic Review, 99, 486–508.

Arrow, K. (1965): "Aspects of the theory of risk-bearing (Yrjo Jahnsson Lectures)," Yrjo Jahnssonin Saatio, Helsinki.

Atkinson, A. B. (1970): "On the Measurement of Inequality," Journal of Economic Theory, 2, 244-263.

Attanasio, O., A. Barr, J. Cardenas, G. Genicot and C. Meghir (2012): "Risk pooling, risk preferences, and social networks," AEJ - Applied Economics, 4, 134-167.

Austen-Smith, D. and R. Fryer (2005): "The economic analysis of acting white," Quarterly Journal of Economics, 120, 551-583.

Bala, V. and S. Goyal (2000): "A noncooperative model of network formation," Econometrica, 68, 1181-1229.

Banerjee, A., A. Chandrasekhar, E. Duflo and M. Jackson (2013a): "The diffusion of microfinance," Science, 341, No. 6144 1236498

Banerjee, A., E. Duflo, M. Ghatak, and J. Lafortune (2013b): "Marry for what? Caste and mate selection in modern India." American Economic Journal: Microeconomics, 5(2): 33-72.

Banerjee, A., A. Chandrasekhar, E. Duflo and M. Jackson (2014a): "Gossip: Identifying central individuals in a social network," MIT and Stanford working paper.

Banerjee, A., A. Chandrasekhar, E. Duflo and M. Jackson (2014b): "Exposure to formal loans: Changes in social networks," MIT and Stanford working paper.

Banerjee, A., E. Breza, E. Duflo and C. Kinnan (2014c): "Does microfinance foster business growth? The importance of entrepreneurial heterogeneity," mimeo MIT.

Bennett, E. (1988): "Consistent Bargaining Conjectures in Marriage and Matching," Journal of Economic Theory, 45, 392-407.

Bergstrom, T. and H. Varian (1985): "When do markets have transferable utility?," Journal of Economic Theory, 35, 222-233.

Billand, P., Bravard, C. and S. Sarangi (2012): "Mutual insurance networks and unequal resource sharing in communities," DIW Berlin Discussion Paper.

Binzel, C., E. Field and R. Pande (2017): "Does the Arrival of a Formal Financial Institution Alter Informal Sharing Arrangements? Experimental Evidence from Village India" mimeo Harvard University.

Bloch, F., G. Genicot, and D. Ray (2008): "Informal insurance in social networks,"Journal of Economic Theory, 143, 36-58.

Borch, K. (1962): "Equilibrium in a reinsurance market," Econometrica, 30, 424-444.

Bramoullé, Y. and R. Kranton (2007a): "Risk-sharing networks,"Journal of Economic Behavior and Organization, 64, 275-294.

Bramoullé, Y. and R. Kranton (2007b): "Risk sharing across communities," American Economic Review Papers & Proceedings, 97, 70.

Brügemann, B., P. Gautier, and G. Menzio (2018a): "Intra Firm Bargaining and Shapley Values," forthcoming Review of Economic Studies.

Brügemann, B., P. Gautier, and G. Menzio (2018b): "Rolodex Game in Networks," mimeo.

Burt, Y. (1992): "Structural holes," Academic Press, New York.

Calvo-Armengol, A. (2001): "Bargaining power in communication networks," Mathematical Social Sciences, 41, 69-87.

Calvo-Armengol, A. (2002): "On bargaining partner selection when communication is restricted," International Journal of Game Theory, 30, 503-515.

Calvo-Armengol, A. and R. Ilkilic (2009): "Pairwise-stability and Nash equilibria in network formation," International Journal of Game Theory, 38, 51-79.

Cameron, A., J. Gelbach and D. Miller (2008): "Bootstrap-based improvements for inference with clustered errors, " The Review of Economics and Statistics, 90(3), 414-427.

A. Chandrasekhar and R. Lewis (2014): "Econometrics of sampled networks," mimeo Stanford University.

Cole, H., and N. Kocherlakota (2001): "Efficient allocations with hidden income and hidden storage," The Review of Economic Studies 68, 523-542.

Coate, S. and M. Ravaillon (1993): "Reciprocity without commitment: Characterization and performance of informal insurance arrangements," Journal of Development Economics, 40, 1–24.

Corominas-Bosch, M. (2004): "Bargaining in a network of buyers and sellers," Journal of Economic Theory, 115, 35-77.

Elliott, M. (2015): "Inefficiencies in networked markets, " AEJ Microeconomics, 7(4), 43-82.

Elliott, M. and F. Nava (2016): "Decentralized Bargaining: Efficiency and the Core," LSE working paper.

Fafchamps, M. (1992): "Solidarity networks in preindustrial societies: Rational peasants with a moral economy," Economic Development and Cultural Change, 41, 147-74.

Fafchamps, M. and F. Gubert (2007): "The formation of risk sharing networks," Journal of Development Economics, 83, 326-350.

Feigenberg, B., E. Field and R. Pande (2013): "The economic returns to social interaction: Experimental evidence from micro finance," Review of Economic Studies, 80, 1459-1483.

Field, E. and R. Pande (2018): "An experimental test of the association between network centrality and cross-village links," mimeo.

Fontenay, C. and J. Gans (2014): "Bilateral bargaining with externalities," Journal of Industrial Economics, 62.4 (2014): 756-788.

Glaeser, E., D. Laibson, and B. Sacerdote (2002): "An economic approach to social capital," The Economic Journal, 112, F437-F458.

Goyal, S. and F. Vega-Redondo (2007): "Structural holes in social networks ," Journal of Economic Theory, 137, 460–492.

Hart, O. and J. Moore (1990): "Property Rights and the Nature of the Firm," Journal of political economy, 98(6), 1119–1158.

Hojman, D. and A. Szeidl (2008): "Core and periphery in networks," Journal of Economic Theory, 139, 295-309.

Jackson, M. (2003): "The stability and efficiency of economic and social networks," in Advances in Economic Design, edited by Murat Sertel and Semih Koray, Springer-Verlag, Heidelberg.

Jackson, M. (2005): "Allocation rules for network games," Games and Economic Behavior, 51, 128-154.

Jackson, M. (2010): "Social and economic networks," Princeton University Press.

Jackson, M., T. Rodriguez-Barraquer and X. Tan (2012): "Social capital and social quilts: Network patterns of favor exchange," The American Economic Review 102.5 (2012): 1857-1897.

Jackson, M. and A. Wolinsky (1996): "A strategic model of social and economic networks," Journal of Economic Theory, 71, 44-74.

Karlan, D., M. Mobius, T. Rosenblat and A. Szeidl (2009): "Trust and social collateral," Quarterly Journal of Economics, 124, 1307-1361.

Kets, W., G. Iyengar, R. Sethi and S. Bowles (2011): "Inequality and network structure," Games and Economic Behavior, 73, 215-226.

Kinnan, C. (2011): "Distinguishing barriers to insurance in Thai villages," mimeo Northwestern University.

Kinnan, C. and R. Townsend (2012): "Kinship and financial networks, formal financial access, and risk reduction," The American Economic Review, 102, 289-293.

Kranton, R. and D. Minehart (2001): "A theory of buyer-seller networks," American Economic Review, 91, 485-508.

Ligon, E., , J. Thomas, and T. Worrall (2002): "Informal insurance arrangements with limited commitment: Theory and evidence from village economies," Review of Economic Studies, 69, 209–244.

Manea, M. (2011): "Bargaining in stationary networks," American Economic Review, 101, 2042-80.

Matsuura, K. and C.J. Willmott (2012): "Terrestrial precipitation: 1900–2008 gridded monthly time series,"Center for Climatic Research Department of Geography Center for Climatic Research, University of Delaware.

Mazzocco, M. and S. Saini (2012): "Testing efficient risk sharing with heterogeneous risk preferences,"The American Economic Review, 102.1, 428-468.

Mehra, A., Kilduff, M. and D. Bass (2001): " The social networks of high and low self-monitors: implications for workplace performance,"Admin. Sci. Quart. 46 (2001) 121–146.

M. Morduch (1991): "Consumption smoothing across space: Tests for village-level responses to risk." Harvard University.

M. Morduch (1999): "Between the state and the market: Can informal insurance patch the safety net?" The World Bank Research Observer 14.2, 187–207.

Munshi, K. and M. Rosenzweig (2006): "Traditional institutions meet the modern world: Caste, gender, and schooling choice in a globalizing economy," The American Economic Review, 1225-1252.

Munshi, K. and M. Rosenzweig (2009): "Why is mobility in India so low? Social insurance, inequality, and growth,"National Bureau of Economic Research, No. w14850.

Myerson, R. (1977): "Graphs and cooperation in games," Mathematics of Operations Research, 2, 225-229.

Myerson, R. (1980): "Conference structures and fair allocation rules," International Journal of Game Theory, 9.3, 169-182.

Myerson, R. (1991): "Game theory: Analysis of conflict, Harvard University Press, Cambridge.

Navarro, N. and A. Perea, (2013): "A simple bargaining procedure for the Myerson value," The BE Journal of Theoretical Economics, 13, 1-20.

Pin, P. (2011): "Eight degrees of separation," Research in Economics, 65, 259-270.

Podolny, J., and J. Baron, (1997): "Resources and relationships: Social networks and mobility in the workplace," American Sociological Review, 673-693.

Pycia, M. (2012): "Stability and preference alignment in matching and coalition formation," Econometrica, 80(1), 323-362.

Rosenzweig, M. (1988): "Risk, implicit contracts and the family in rural areas of low-income countries,"Economic Journal, 98, 1148-70.

Rosenzweig, M. and O. Stark (1989): "Consumption smoothing, migration, and marriage: Evidence from rural India,"Journal of Political Economy, 97(4), 905-926.

Slikker, M. (2007): “Bidding for surplus in network allocation problems,” Journal of Economic Theory, 137, 493-511.

Srinivas, M. (1962): “Caste in modern India and other essays,” Bombay, India: Asia Publishing House.

Stole, L. and J. Zwiebel (1996): “Intra-firm bargaining under non-binding contracts,” Review of Economic Studies, 63, 375-410.

Townsend, R. (1994): “Risk and insurance in village India,” Econometrica, 62, 539-591.

Udry, C. (1994): “Risk and insurance in a rural credit market: An empirical investigation in Northern Nigeria,” Review of Economic Studies, 61, 495-526.

van Lint, J. H., and R. Wilson (2001): “A course in combinatorics,” Cambridge university press.

Walker, T. and J. Ryan (1990): “Village and household economics in India’s semi-arid tropics,” Johns Hopkins University Press, Baltimore.

Wilson, R. (1968): “The theory of syndicates,” Econometrica, 36, 119-132.

Zahl, S. (1963): “An Allocation Problem With Applications to Operations Research and Statistics,” Operations Research, 11, 426-441.

Appendix I. Omitted Proofs

Proof of Proposition 1. To prove the first statement, consider villagers' certainty-equivalent consumption. Let $\hat{K}$ be some constant, and consider the certain transfer K' (made in all states of the world) that i requires to compensate him for keeping a stochastic consumption stream $c_i + \hat{K}$ instead of another stochastic consumption stream $c_i' + \hat{K}$:

$$\begin{aligned}
\mathbf{E}[v(c_i + \hat{K} + K')] &= \mathbf{E}[v(c_i' + \hat{K})] \\
-\frac{1}{\lambda} e^{-\lambda \hat{K}} e^{-\lambda K'} \mathbf{E}[e^{-\lambda c_i}] &= -\frac{1}{\lambda} e^{-\lambda \hat{K}} \mathbf{E}[e^{-\lambda c_i'}] \\
e^{\lambda K'} &= \frac{\mathbf{E}[e^{-\lambda c_i}]}{\mathbf{E}[e^{-\lambda c_i'}]} \\
K' &= \frac{1}{\lambda} \left(\ln \left(\mathbf{E}[e^{-\lambda c_i}] \right) - \ln \left(\mathbf{E}[e^{-\lambda c_i'}] \right) \right) \qquad (16)
\end{aligned}$$

This shows that the amount K' needed to compensate i for taking the stochastic consumption stream $c_i + \hat{K}$ instead of $c_i' + \hat{K}$ is independent of $\hat{K}$. As a villager's certainty-equivalent consumption for a lottery is independent of his consumption level, certainty-equivalent units can be transferred among the villagers without affecting their risk preferences, and expected utility is transferable.

Next, we characterize the set of Pareto efficient risk sharing agreements. Borch (1962) and Wilson (1968) showed that a necessary and sufficient condition for a risk-sharing arrangement between i and j to be Pareto efficient is that in almost all states of the world $\omega \in \Omega := \mathbf{R}^{|S|}$,

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)} \right) \Big/ \left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)} \right) = \alpha_{ij} \qquad (17)$$

where α_{ij} is a constant. Substituting in the CARA utility functions, this implies that

$$\begin{aligned}
\frac{e^{-\lambda c_i(\omega)}}{e^{-\lambda c_j(\omega)}} &= \alpha_{ij} \\
c_i(\omega) - c_j(\omega) &= -\frac{\ln(\alpha_{ij})}{\lambda} \\
\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] &= -\frac{\ln(\alpha_{ij})}{\lambda} \\
c_i(\omega) - c_j(\omega) &= \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] \qquad (18)
\end{aligned}$$

Letting i and j be neighbors such that $j \in \mathbf{N}(i)$, equation 18 means that when i and j reach any Pareto-efficient risk-sharing arrangement their consumptions will differ by the same constant in all states of the world. Moreover, by induction the same must be true for all pairs of path-connected villagers.

Consider now the problem of splitting the incomes of a set of villagers $\mathbf{S}$ in each state of the world to minimize the sum of their consumption variances:

$$\min_{\mathbf{c}} \sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) \qquad \text{subject to} \quad \sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega) \quad \text{for all } \omega. \tag{19}$$

If we denote a CDF of income probability distribution on $\Omega = \mathbf{R}^{|S|}$ by $F(\cdot)$,

$$\sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) = \int_\Omega \sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 dF(\omega). \tag{20}$$

Since $\mathrm{Var}(c_i(\omega) + a_i) = \mathrm{Var}(c_i(\omega))$, the sum of variances is invariant to state-independent changes in a consumption profile, and the variance-minimizing consumption profile exists for any profile of expected consumptions $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$: $\sum_{i\in\mathbf{S}} \mathbf{E}[c_i] = \sum_{i\in\mathbf{S}} \mathbf{E}[y_i]$. Fix any such profile of expected consumptions, $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$. Similarly to Wilson (1968), we apply Theorem 1 from Zahl (1963) to our minimization problem. We denote a Lagrange multiplier attached to constraint $\sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega)$ by $\gamma(\omega)$. Then, the corresponding Lagrangian of the problem is

$$\int_\Omega \left[\sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 - \gamma(\omega) \sum_{i\in\mathbf{S}} c_i(\omega) \right] dF(\omega).$$

By pointwise minimization with respect to $c_i(\omega)$ we obtain that for each $i \in \mathbf{S}$ and almost every $\omega \in \Omega$, $2(c_i^*(\omega) - \mathbf{E}[c_i]) = \gamma(\omega)$. Thus, $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for all $i, j \in \mathbf{S}$. Note that this equality as well implies that $\mathbf{E}[c_i^*(\omega)] = \mathbf{E}[c_i]$, and $\{c_i^*(\omega)\}$ indeed solves the minimization problem. Thus, the condition $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for almost all ω is exactly the same as the necessary and sufficient condition for an ex-ante Pareto efficiency. Hence, a risk-sharing agreement is Pareto efficient if and only if the sum of the consumption variances for all path-connected villagers is minimized.

Using the necessary and sufficient condition for efficient risk sharing, we obtain

$$\sum_{k\in\mathbf{S}} y_k(\omega) = \sum_{k\in\mathbf{S}} c_k(\omega) = |\mathbf{S}| c_i(\omega) - \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_k(\omega)] \right), \tag{21}$$

which implies that

$$c_i(\omega) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] \right) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \tau_i, \tag{22}$$

where $\tau_i = \mathbf{E}[c_i(\omega)] - \mathbf{E}[\frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega)]$. □

Proof of Lemma 4. Agent i's net benefit from forming link l_{ij} is $(MV_i(L) - MV_i(L \setminus \{l_{ij}\}) - \kappa_w)$. We need to show that

$$MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) = md(i, j, L)V. \tag{23}$$

Some additional notation will be helpful. Suppose agents arrive in a random order, with a uniform distribution on all possible arrival orders. The random variable $\widehat{\mathbf{S}}_i \subseteq \mathbf{N}$ identifies the set of agents, including i, who arrive weakly before i. For each arrival order, we then have an associate network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ that describes the network formed upon i's arrival (the subnetwork of L induced by agents $\widehat{\mathbf{S}}_i$). Let $q(i,j,L)$ be the probability that i and j are path-connected on network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$.

The certainty-equivalent value of the reduction in variance due to a link l_{ij} in a graph $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ is V if the link is essential and 0 otherwise. The change in i's Myerson value, $MV_i(L) - MV_i(L \setminus \{l_{ij}\})$, is then $(q(i,j,L) - q(i,j,L \setminus \{l_{ij}\}))\, V$. However, $q(i,j,L) = 1/2$. To see this, note that $l_{ij} \in L$ and therefore in every order of arrival in which i arrives after j (which happens with probability 1/2), i and j are path-connected on the network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$, while i and j are never path-connected on $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ when j arrives after i.

Probability $q(i,j,L \setminus \{l_{ij}\})$ can be computed by the inclusion-exclusion principle, using the fact that the probability of a path connecting i and j existing on network $\mathcal{L}_{L\setminus\{l_{ij}\}}(\widehat{\mathbf{S}}_i)$ is equal to the probability that for some path connecting i and j on $L \setminus \{l_{ij}\}$ all agents on the path are present in $\widehat{\mathbf{S}}_i$. Thus

$$q(i,j,L \setminus l_{ij}) = \sum_{k=1}^{|\mathbf{P}(i,j,L\setminus l_{ij})|} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le |\mathbf{P}(i,j,L)|} \left(\frac{1}{|P_{i1} \cup \cdots \cup P_{ik}|} \right) \right). \tag{24}$$

We therefore have that

$$MV_i(L) - MV_i(L \setminus l_{ij}) = (1/2 - q(i,j,L \setminus l_{ij}))V = md(i,j,L)V, \tag{25}$$

where the last equality follows from the definition of Myerson distance.

□

Proof of Proposition 5. For there to be underinvestment in a pairwise-stable network L, there must exist a link $l_{ij} \notin L$ for which $TS(L \cup l_{ij}) - TS(L) > 2\kappa_w$. This can only happen if l_{ij} is essential on $L \cup l_{ij}$ as otherwise $TS(L \cup l_{ij}) - TS(L) = 0$. Thus $TS(L \cup l_{ij}) - TS(L) = V$ and so $V > 2\kappa_w$. As $l_{ij} \notin L$ and L is pairwise stable, Lemma 4 implies that $md(i,j,L) \le \kappa_w/V$. However, as l_{ij} is essential on $L \cup l_{ij}$, $md(i,j,L) = 1/2$. Substituting this into the condition from Lemma 4 we get $V \le 2\kappa_w$, leading to a contradiction.

For the second part of the proposition, let L be a pairwise-stable network and let $l_{ij} \in L$ be an essential link on L in which there is overinvestment. Thus $TS(L) - TS(L \setminus \{l_{ij}\}) = V < 2\kappa_w$. Since l_{ij} is essential, $md(i,j,L \setminus \{l_{ij}\}) = 1/2$. But Lemma 4 implies that $md(i,j,L \setminus \{l_{ij}\}) \ge \kappa_w/V$. We therefore have that $V \ge 2\kappa_w$, leading to a contradiction. □

Proof of Proposition 6.

Part (i): By remark 3 and under our regularity condition, all efficient networks are tree networks. By definition, in all tree networks any pair of agents i and j have a unique path between them. Thus, for a tree network L with diameter $d(L)$, there exist agents i and j with a unique path between them of length $d(L)$ and all other pairs of agents have a weakly shorter path between them. Thus by equation 11:

$$md(i,j,L) = \frac{1}{2} - \frac{1}{d(L)} \geq md(k,k',L) \qquad \text{for all } k,k' \in \mathbf{N}. \tag{26}$$

By Proposition 5 there is no underinvestment in any stable network. Lemma 4 therefore implies that the efficient network L is stable if and only if $md(k,k',L) \leq \kappa_w/V$ for all k,k' such that $l_{kk'} \notin L$. As $md(i,j,L) \geq md(k,k',L)$ and $md(i,j,L) = 1/2 - 1/d(L)$ (see equation 26), this condition simplifies and the efficient network L is stable if and only if

$$\frac{V - 2\kappa_w}{V} \leq \left(\frac{2}{d(L)}\right). \tag{27}$$

As $d(L)$ gets large, the right-hand side converges from above to 0 and so in the limit, the condition for stability becomes $V \leq 2\kappa_w$, which is violated by our regularity condition. Thus, there exists a finite $\overline{d}(L)$ such that the efficient network L is stable if and only if $d(L) \leq \overline{d}(L)$.

Rearranging equation 27, L is stable if and only if

$$d(L) \leq 2\left(\frac{V}{V - 2\kappa_w}\right). \tag{28}$$

So the key threshold is $\overline{d}(\kappa_w) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.

Fixing the number of agents $|\mathbf{N}|$ in an efficient (tree) network L, the star network is the unique (tree) network (up to a relabeling of players) that minimizes the diameter $d(L)$ while the line network is the unique (tree) network (up to a relabeling of players) that maximizes the diameter $d(L)$. The result now follows immediately.

Part (ii): On any efficient networks all links are essential and generate a net surplus of $V - 2\kappa_w > 0$, where the inequality follows from our regularity condition. As i and j must benefit equally at the margin from the link l_{ij} (see condition (ii) in the definition of agreements that are robust to split-the-difference renegotiation), agent i's expected payoff on an efficient network L is

$$u_i(L) = |\mathbf{N}(i;L)|(V/2 - \kappa_w) > 0. \tag{29}$$

Thus i's net payoff is proportional to his degree.

For any tree network L other than the star network let agent k be one of the agents with the highest degree. Consider a link $l_{ij} \in L$ such that $i,j \neq k$. As L is a tree there is a unique path from i to k and a unique path length from j to k. As we are on a tree network, either the path from j to k passes through i, or else the path from i to k passes through j. Hence either i or j is closer to k and without loss of generality we let i have a longer path to k than j. We now delete the link l_{ij} and replace it with the link l_{ik}. This operation generates a new tree network. Moreover, repeating this operations until there are no links l_{ij} such that $i, j \neq k$, defines an algorithm.

This algorithm terminates at star networks as the operation cannot be applied to this network; There are no links of l_{ij} such that $i, j \neq k$. Moreover the operation can be applied to any other tree network because on all other tree networks there exists an l_{ij} such that $i, j \neq k$. Finally, in each step of the algorithm the degree of k increases and so the algorithm must terminate in a finite number of steps. Moreover, the algorithm must terminates at the star network with k at the center.

By construction, at each step of the above algorithm we decrease the degree of some agent $j \neq k$ and increase the degree of k. Suppose we start with a network L and consider a step of this rewiring where the link l_{ij} is deleted and replaced by the link l_{ik}. Only the expected payoff of agents j and k on L and $L \cup l_{ik} \setminus l_{ij}$ change; The degrees of all other agents remain constant and thus by equation 29 so do their payoffs. Letting $\alpha = (V/2 - \kappa_w)$, we have $u_j(L) = \alpha d_j(L)$, $u_k(L) = \alpha d_k(L)$, $u_j(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_j(L) - 1)$ and $u_k(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_k(L) + 1)$.

It follows that welfare $W(u) = \sum_i f(u_i)$ (see equation 9) decreases through the rewiring in this step if and only if

$$f(\alpha(d_j - 1)) + f(\alpha(d_k + 1)) - f(\alpha d_j) - f(\alpha d_k) < 0, \tag{30}$$

which is equivalent to:

$$f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1)) \tag{31}$$

As $f(\cdot)$ is increasing, strictly concave and differentiable $f'(\alpha d_j)\alpha < f(\alpha d_j) - f(\alpha(d_j - 1))$ and $f'(\alpha d_k)\alpha > f(\alpha(d_k + 1)) - f(\alpha d_k)$. Moreover, by concavity $f'(\alpha d_j) \geq f'(\alpha d_k)$ (as $d_k \geq d_j$). Combining these inequalities establishes the claim that $f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1))$.

Thus at each step of the rewiring welfare $W(u)$ decreases. For each network L' reached during the algorithm we can consider the average expected utility $u'(L')$ which if distributed equally would generate the same level of welfare as obtained on L. As aggregate welfare is decreasing at each step of the rewiring $u'(L)$ must be decreasing too. However, the total surplus generated by risk sharing remains constant and so average expected utility $\overline{u}$ remains constant. Recall that Atkinson's inequality measure / index is given by $I(L) = (1 - (u'(L)/\overline{u})$. Thus at each step of the rewiring the inequality measure $I(L)$ increases. As this rewiring can be used to move from any tree network to the star network, stars network and only star networks maximize inequality among the set of tree networks, which correspond to the set of efficient networks under our regularity condition. As this argument holds for any strictly increasing and differentiable, concave function f it holds for all inequality measures in the Atkinson class.

Consider now an alternative rewiring of a tree network L. Let k be one of the agents with highest degree on L and let j be one of the agents with degree 1 on L. As tree networks contain no cycles, there always exists agents with degree 1 (leaf agents). Pick one of k's neighbors $i \in \mathbf{N}(k; L)$, remove the link l_{ik} from L and add the link l_{ij} to L. This operation generates a new tree network. Repeating this operation until the highest degree agent has degree 2 defines an algorithm. As the unique tree network with a highest degree of 2 is the line network, the algorithm terminates at line networks and only line networks. At each stage of the rewiring we either reduce the degree of the highest degree agent k or reduce the number of agents who have the highest degree. Thus the algorithm must terminate in a finite number of steps at a line network. Moreover, reversing the argument above, inequality is reduced at each step of the rewiring for any inequality measure in the Atkinson class.

□

Proof of Proposition 7. By definition, underinvestment within group for a network L requires that there exists an $l_{ij} \notin L$ such that $G(i) = G(j)$ and for which $TS(L \cup l_{ij}) - TS(L) > 2\kappa_w$. As $TS(L \cup l_{ij}) - TS(L) = 0$ for all non-essential links, l_{ij} must be essential on $L \cup \{l_{ij}\}$. Thus l_{ij} is also essential on $\widehat{L} \cup \{l_{ij}\}$ for any $\widehat{L} \subseteq L$. Equation 13 then implies that $TS(\widehat{L} \cup l_{ij}) - TS(\widehat{L}) \geq V$ for any $\widehat{L} \subseteq L$.

Consider any arrival order in which i arrives after j and let $\mathbf{S}_i$ be the agents that arrive (strictly) before i. Agent i's marginal contribution to total surplus without l_{ij} when i arrives is then $TS(L(\mathbf{S}_i \cup \{i\})) - TS(L(\mathbf{S}_i))$ while with l_{ij} it is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i))$. So i's additional marginal contribution to total surplus when l_{ij} has been formed is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\}))$. As $L(\mathbf{S}_i \cup \{i\}) \subseteq L$, by the above argument $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\})) \geq V$. As i arrives after j in half the arrival orders, i's average additional incremental contribution to total surplus when l_{ij} has been formed is at least $V/2$. Thus $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \geq V/2$. An equivalent argument establishes that $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \geq V/2$. Under our regularity condition $V/2 > \kappa_w$ and so i and j have a profitable deviation to form l_{ij} and the network L is not stable. As L was an arbitrary network within underinvestment within group, there is no stable network with underinvestment within group. □

Proof of Proposition 8. The proof of the first part of the statement has four steps.

Step 1: Consider any efficient network L that is robust to overinvestment inefficiency within group. This implies that for all path-connected agents i, j such that $G(i) = G(j)$ and $l_{ij} \notin L$, either $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ or $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$. However, by condition (i) in the definition of agreements that are robust to split-the-difference renegotiation, $MV_i(L \cup \{l_{ij}\}) - MV_i(L) = MV_j(L \cup \{l_{ij}\}) - MV_j(L)$ and so both $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ and $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$.

Step 2: Let a network $\widehat{L} := \{l_{ij} : G(i) = G(j), l_{ij} \in L\}$ be a network formed from L by deleting all across-group links. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. As the network L is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between i and j cannot contain an across-group link. So, i is path-connected to j on the induced subnetwork $L(\mathbf{S})$ if and only if i is path-connected to j on the induced subnetwork $\widehat{L}(\mathbf{S})$. Thus, by equation 13, the additional variance reduction that i and j can now achieve by forming a superfluous across-group link on $\widehat{L}(\mathbf{S})$ is weakly lower than on $L(\mathbf{S})$. So, by the Myerson value definition (equation 6), $MV_i(\widehat{L}\cup\{l_{ij}\}) - MV_i(\widehat{L}) \leq MV_i(L\cup\{l_{ij}\}) - MV_i(L)$ and $MV_j(\widehat{L}\cup\{l_{ij}\}) - MV_j(\widehat{L}) \leq MV_i(L\cup\{l_{ij}\}) - MV_i(L)$. This implies that $\widehat{L}$ is robust to overinvestment within group.

Step 3: Let a network $\widehat{L}'$ be a network formed from $\widehat{L}$ by rewiring (alternately deleting then adding a link) each within-group network into a star (for an algorithm that does this, see the part (ii) of the proof of Proposition 6). Consider any two agents i', j' such that $G(i') = G(j')$, $l_{i'j'} \notin \widehat{L}'$. By part (i) of Proposition 6, $MV_{i'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{i'}(\widehat{L}') \leq MV_i(\widehat{L} \cup \{l_{ij}\}) - MV_i(\widehat{L})$ and $MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}') \leq MV_j(\widehat{L} \cup \{l_{ij}\}) - MV_j(\widehat{L})$. Thus $\widehat{L}'$ is robust to overinvestment within group.

Step 4: Finally, consider any network $L' \in \mathcal{L}^{CSS}$. This network can be formed by adding a set of across-group links to a network $\widehat{L}'$ such that $\widehat{L}' \subseteq L'$ and if $l_{kk'} \in L' \setminus \widehat{L}'$ then $G(k) \neq G(k')$. Consider any subset of agents $\mathbf{S}' \subseteq \mathbf{N}$ such that $i', j' \in \mathbf{S}'$. Recall that $G(i') = G(j')$ and note that by the construction of L', $l_{i'j'} \notin L'$. On the induced subnetwork $L'(\mathbf{S}')$, either i' is path-connected to j', in which case $l_{i'j'}$ would be superfluous if added, or else i' and j' are isolated nodes. This is because the within-group network structure for group $G(i')$ is a star. Thus, whenever $l_{i'j'}$ would not be superfluous, the change in i' and j''s Myerson value if it were added is independent of the across-group links that are present: $MV_{i'}(L'\cup\{l_{i'j'}\}) - MV_{i'}(L') = MV_{i'}(\widehat{L}'\cup\{l_{i'j'}\}) - MV_{i'}(\widehat{L}')$ and $MV_{j'}(L'\cup\{l_{i'j'}\}) - MV_{j'}(L') \leq MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}')$. Thus L' is robust to overinvestment within group.

We turn now to the second part of the result. If $L \notin \mathcal{L}^{CCS}$, then there will be agents i, j such that $G(i) = G(j)$ and $l_{ij} \notin L$ such that either the within-group network structure for $G(i)$ is not a star, or else it is a star but there are across-group links being held by an agent who is not the center agent. In the first case, the inequality in step 3 will be strict by Proposition 6. In the second case, we can without loss of generality let agent i be the non-center agent holding the across-group link. Then, by equation 13, the inequality in step 2 will be strict. Thus for some parameter values L will not be robust to overinvestment within group, but L' will be. □

Proof of Lemma 9. Denote the set of all possible arrival orders for the set of agents $\mathbf{N}$, by $\mathcal{A}(\mathbf{N})$. Order this set of $|\mathbf{N}|!$ arrival orders in any way, denoting the kth arrival order by $\widehat{A}_k \in \mathcal{A}(\mathbf{N})$. We will then construct an alternative ordering, in which we denote the kth arrival order by $\widetilde{A}_k \in \mathcal{A}(\mathbf{N})$, such that for arrival order $\widetilde{A}_k$,

(i) i arrives at the same time as agent i' does for the arrival order $\widehat{A}_k$;

(ii) when i arrives he connects to exactly the same set of agents from $\mathbf{N} \setminus \mathbf{S}_0$ that i' connects to upon his arrival for the arrival order $\widehat{A}_k$;

(iii) when i arrives he connects to weakly more agents from $\mathbf{S}_0$ that i' connects to upon his arrival for the arrival order $\widehat{A}_k$.

Equation 15 shows that the risk reduction, and hence the marginal contribution made by an agent $k \in \mathbf{S}_0$ from providing the across-group link l_{kj}, is an increasing function of the component size of k's groups. It then follows that

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L). \tag{32}$$

To construct the alternative ordering of the set $\mathcal{A}(N)$ as claimed we will directly adjust individual arrival orders, but in a way that preserves the set $\mathcal{A}(N)$. First, for each arrival order, we switch the arrival positions of i' and i. This alone is enough to ensure that conditions (i) and (ii) are satisfied. There are $|\mathbf{S}_0|!$ possible arrival orders for the set of agents $\mathbf{S}_0$. Ignoring for now the other agents, we label these arrival orders lexicographically. First we order them, in ascending order, by when i arrives. Next, we order them in ascending order by the number of agents i is connected to upon his arrival. Breaking remaining ties in any way, we have labels $1_i, 2_i, \ldots, |\mathbf{S}_0|!_i$. We then let every element of $\mathcal{A}(N)$ inherit these labels, so that two arrival orders receive the same label if and only if the agents $\mathbf{S}_0$ arrive in the same order. We now construct a second set of labels by doing the same exercise for i', and denote these labels by $1_{i'}, 2_{i'}, \ldots, |\mathbf{S}_0|!_{i'}$. We are now ready to make our final adjustment to the arrival orders. For each original arrival order $\widehat{A}_k$ we find the associated (second) label. Suppose this is $x_{i'}$. We then take the current kth arrival order (given the previous adjustment), and reorder (only) the agents in $\mathbf{S}_0$, so that the newly constructed arrival order now has (first) label x_i. Because of the lexicographic construction of the labels, the arrival position of agent i will not change as a result of this reordering of the arrival positions of agents in $\mathbf{S}_0$, so conditions (i) and (ii) are still satisfied. In addition, condition (iii) will now be satisfied from the definition of i being more central than i'. The only remaining thing to verify is that the set of arrival orders we are considering has not changed (i.e. that we have, as claimed, constructed an alternative ordering of the set $\mathcal{A}(N)$) and this also holds by construction. $\square$

Proof of Proposition 10. Let L be an efficient network that is robust to underinvestment across group. This implies that for any across-group link $l_{ij} \in L$ between groups $g = G(i)$ and $\hat{g} = G(j) \neq g$, $MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \geq \kappa_a$, where the inequality follows from condition (i) in the robustness to split-the-difference renegotiations definition.

We now rewrite L. As the network L is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between any two agents from the same group cannot contain an across-group link. We can therefore rewrite the within-group network structures of L to obtain a star by sequentially deleting and then adding within-group links (an algorithm that does this is presented in the proof of part (ii) of Proposition 6). Do this rewiring so that agent i is the agent at the center of the within-group network for group $G(i)$ and let j be the agent at the center of the within-group network for group $G(j)$. Finally, we rewire across-group links so that the same groups remain directly connected, but all across-group links are held by the center agents. Let the network obtained be L'. By construction, $L' \in \mathcal{L}^{CCS}$.

Under our definition of Myerson centrality, it is staightforward to verify that both i and j are weakly more Myerson central within their respective groups on network L' than on network L. An argument almost identical to that in the proof of Lemma 9 then implies that i' and j' have better incentives to keep the link $l_{i'j'}$ than i and j have to keep the link l_{ij} (because the argument is more or less identical we skip it). Hence,

$$MV_{i'}(L) - MV_{i'}(L \setminus \{l_{i'j'}\}) \geq MV_i(L) - MV_i(L \setminus \{l_{ij}\}) \tag{33}$$

$$MV_{j'}(L) - MV_{j'}(L \setminus \{l_{i'j'}\}) \geq MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \tag{34}$$

Network L' is therefore robust to underinvestment. Moreover, whenever the within-group networks of i and j on network L are not both stars with i and j at the centers, the inequality is strict because both i and j are strictly more Myerson central within-group on L' than on L. There then exists a range of parameter specifications for which any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but L is not. □

Appendix II. Permitting some free links

In our model we assume that each link costs a fixed amount to form, but in practice, certain relationships will already exist permitting risk sharing without any investment. We now permit this possibility by assuming that there are a set of within-group links that can be formed for free. These links might represent family relationships or close friendships formed in childhood. Within this context we re-examine the structure of efficient networks, and those networks that are most stable to underinvestment and overinvestment.

More formally, we let $\widehat{L}$ denote the set of within group links that can be formed for free. As, by the Myerson Value calculation, a link strictly increases the expected utility an agent receives in a risk sharing arrangement, we let all such links be always formed. The network $\widehat{L}$ will consist of a set of components, each of which contains agents from the same group. For each such component C, we identify an agent $i^*(C) \in \text{argmin}_i \max_j md_{ij}(C)$. This is agent who has the lowest maximum Myerson Distance to any other agent in the component C. We will refer to agent $i^*(C)$ as the Myerson distance central agent in component C and let C_i denote the component to which i belongs. Considering all components, we then have a set of Myerson distance central agents $I^* = (i^*(C))_C$. Finally, we identify a Myerson distance central agent associated with the largest distance, $i^{**} \in \text{argmax}_{i^* \in I^*} \max_{j \in C_{i^*}} md_{i^*j}$.

When there is one group, we dub a network generated by forming all free links, and the links $l_{i^*i^{**}}$ for all $i^* \neq i^{**}$ a *central connections network*.

Suppose there are k different groups and $k' \geq k$ initial components. The set of efficient network then comprises of the set of networks in which all free links are formed, $k-1$ across group links are formed and $k'-k$ within group links are formed, such that a there is a single component. This is the lowest cost way to form a single component, and by assumption it is efficient for all agents to risk share with each other.[46] We now consider those efficient networks that are most robust to underinvestment. When there is one group, central connections networks are always efficient. They are are also most stable within the class of efficient networks.

Proposition 11. *Suppose there is one group. If any efficient network is stable, then all central connections networks are also stable.*

Proof. Consider two components C and C'. For two agents i, j in component C, recall that $md(i, j, C)$ equals 1/2 less the probability that a path exists between i and j on C upon the arrival of i. Suppose now we take two components C and C'. Let agents i, k be in component C and agents j, k' be in component C', and form the bridging link $l_{kk'}$. The probability a path exists between i and j upon i's arrival is now is equal to the probability that a path exists between i and k on C multiplied by the probability that a path exists between k' and j on C'. This is because these events are independent, and when both path exist agents k and k' must have arrived before i and so the link $l_{kk'}$ must be present. It follows that

$$\underset{i,j}{\text{argmax}}\, md_{ij}(C \cup C' \cup \{l_{kk'}\}) = \{i, j : i \in \underset{l}{\text{argmax}}\, md_{lk}(C), j \in \underset{l}{\text{argmax}}\, md_{lk'}(C')\}.$$

Thus the network generated by forming all free links, and the links $l_{i^*i^{**}}$ for all $i^* \neq i^{**}$ minimizes the maximum Myerson distance on an efficient network and, by Lemma 4, is stable if any other efficient network is stable. □

Proposition 11 shows that when some within-group are formed for free, the most stable efficient network continues to create additional links that increase the centrality of the most central agents.

When there are multiple groups, central connections networks within group with the agent i^{**} providing the across group link(s) continue to work well. With multiple groups, agents' incentives to form superfluous within-group links depend on two things. First, as before, whether the link will be essential for a random arrival order, and second, unlike before,

[46]As before, the same set of risk sharing arrangements can be implemented on any given component, and as expected utility is transferable, given that formation costs have been minimized, any point on the Pareto frontier can be obtained.

how many agents from other groups the link provides access to upon i's arrival when it is essential. Incentives to form a superfluous within-group links are increasing in the number of agents from other groups the link provides access to, and decreasing in the number of agents within-group the link provides access to. These considerations make superfluous links to the agent providing the across group link(s) particularly valuable. However, by construction the network generated by forming a central connections network within-group, with the agent i^{**} providing the across group link(s), minimizes the maximum probability that a superfluous link to the agent providing the across group link(s) will be essential for a random arrival order. It thus minimizes the maximum incentives for an agent to form a superfluous link within-group to the agent providing the across group link(s).

Considering the incentives within a group to efficiently form an across-group essential link, a central connections networks within-group is also likely to do well. By Lemma 9 more Myerson central agents have better incentives to form across group links. While central connections networks maximize a slightly different notion of the centrality of the most central agent, in this case agent i^{**}, these measures of centrality are likely to be highly correlated. We therefore expect central connections networks within-group to provide relatively good incentives for across group links to be formed