Information Revelation and Coordination Using Cheap Talk in a Game with Two-Sided Private Information*

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Abstract

We consider a version of the Battle of the Sexes with private information, in which each player has two types and allow cheap talk regarding the players' types before the game. We prove that the unique fully revealing symmetric cheap talk equilibrium exists and has a desirable *type-coordination* property: it fully coordinates on the ex-post efficient pure Nash equilibrium when the players' types are different. Type-coordination is also obtained in a partially revealing equilibrium (in which only one type is not truthful) that exists when the fully revealing equilibrium does not. We show that these cheap talk equilibria are more efficient than the Bayesian-Nash equilibrium of the game without communication. We also prove that there is no meaningful cheap talk equilibrium involving truth-telling if only one player talks.

Keywords: Battle of the Sexes, Private Information, Cheap Talk, Coordination, Full Revelation.

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1 INTRODUCTION

Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework whereby one player has private information but takes no action and the other player is uninformed but is responsible for taking a payoff-relevant decision. There indeed is a small but growing literature on games where both players have private information and can send cheap talk messages to each other.[1] Our aim in this paper is to contribute to this literature by analysing symmetric cheap talk equilibria in a game with two-sided information and two-sided cheap talk.

We analyse a version of the Battle of the Sexes with two-sided private information using unmediated, cheap-talk. The complete information BoS has many economic applications (see the Introduction in Cabrales *et al*, 2000); the corresponding game of incomplete information is not just a natural extension but is also relevant in many of these economic situations where the intensity of preference and its prior probability are important factors. BoS type games may be more complicated with incomplete information, where each player has private information about the "intensity of preference" for the other player's favorite outcome. Apart from its applications, the BoS with private information is clearly of interest to theorists and experimentalists. It is not obvious at all whether truthful revelation and thereby separation of players' types can be achieved in a cheap talk equilibrium for the BoS with private information; moreover, it is also not clear whether coordination using cheap talk, as in the theoretical and the experimental literature with the complete information BoS,[2] would extend to the BoS with private information.

To analyse the above two issues, namely truthful revelation and coordination, we use the simplest possible version of the BoS, as in Banks and Calvert (1992), with two types ("High" and "Low") for each player regarding the payoff from the other player's favorite outcome. The structure of the game we consider here has an in-built tension for each player between the desire to compromise in order to avoid miscoordination and the desire to force coordination on one's preferred Nash equilibrium

[1] Examples of information transmission using two-sided cheap talk under two-sided incomplete information can be found in Farrell and Gibbons (1989), Matthews and Postlewaite (1989), Baliga and Morris (2002), Doraszelski *et al* (2003), Baliga and Sjöström (2004), Chen (2009), Goltsman and Pavlov (2014) and Horner *et al* (2015). Two-sided cheap talk using multiple stages of communication where only one of the players has incomplete information has also been studied by Aumann and Hart (2003) and Krishna and Morgan (2004).

[2] In a seminal paper, Farrell (1987) showed that rounds of cheap talk regarding the intended choice of play reduces the probability of miscoordination; the probability of coordination on one of the two pure Nash equilibria increases with the number of rounds of communication (although, at the limit, may be bounded away from 1). Park (2002) identified conditions for achieving efficiency and coordination in a similar game with three players. Parallel to the theory, the experimental literature also shows that cheap-talk and any pre-play non-binding communication can significantly improve coordination in games like BoS (Cooper *et al*, 1989; Crawford, 1998; Costa-Gomes, 2002; Camerer, 2003; Burton *et al*, 2005).

outcome. This contrasts with the Hawk-Dove game studied in Baliga and Sjöström (2012) and the Cournot game in Goltsman and Pavlov (2014) where a player's preference over the other player's action does not depend on his type or action. The question we ask is whether, in this game, players will (fully or partially) reveal their types in a direct cheap talk equilibrium and also coordinate on Nash equilibrium outcomes in different states of the world. With incomplete information, efficiency and coordination do not necessarily go together; however, one might find it desirable to coordinate on the (ex-post) efficient outcome when the two players are of different types, in which the compromise is made by the player who suffers a smaller loss in utility. In the game we consider here, it is not apriori clear at all whether either full information revelation or the desirable coordination can be achieved in equilibrium.

The primary focus of Banks and Calvert (1992) was to study communication in a similar game using an impartial mediator and the efficiency implications of such mediated communication. Although incentive compatible mediated mechanisms (as in Banks and Calvert, 1992) inform us about all achievable possibilities with strategic communication, it might be impractical to conceive of or employ an impartial mediator in a real-world situation. For instance, in a market entry game (as in Dixit and Shapiro, 1985) or in the adoption of product compatibility standards (as in Farrell and Saloner, 1988), it is not clear how a mechanism involving an impartial mediator can be implemented. However, we know that firms talk to each other and/or make public announcements from time to time. We believe that direct cheap talk communication among players might occur more naturally in a strategic situation and this is the motivation for studying cheap talk equilibria in this paper.

Banks and Calvert (1992) also studied unmediated communication in a similar game allowing more general message spaces (i.e., not restricted to only two types) in the communication phase. Banks and Calvert identified conditions (Proposition 2, Section 4 in their paper) under which the outcome of an ex-ante efficient incentive compatible mediated mechanism can be achieved as the equilibrium of an unmediated communication process. In contrast, the focus of our current paper is to identify conditions under which (full) revelation occurs at the cheap talk stage and some form of coordination property holds. Obviously, these objectives are different from those studied in Banks and Calvert (1992). Ganguly and Ray (2009) have analysed this game to see if a truthful cheap talk equilibrium, in which the players reveal their types truthfully before playing, exists at all[3] and compared it with the mediated equilibrium of Banks and Calvert (1992).

The main contributions of this paper are thus two-fold. We first prove that there exists a unique fully revealing symmetric cheap talk equilibrium of this game in which the players announce their types truthfully (Theorem 1).[4] Theorem 1 also suggests that full revelation is not a cheap talk equilibrium

[3] We actually correct their result in this paper.

[4] Considering symmetric communication processes seems reasonable here; since the players are identical, facing an

when the probability of a player being High-type is too high or too low; the allowable range of the prior probability of the High-type for the fully revealing equilibrium to exist in Theorem 1 has to be moderately low with the upper bound being strictly less than $\frac{1}{2}$. Secondly, we note that our unique fully revealing cheap talk equilibrium has the desirable *type-coordination property*: when the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium.

We then consider partially revealing equilibria particularly for situations when the fully revealing equilibrium does not exist. Keeping the spirit of the fully revealing equilibrium, we characterise a class of partially revealing cheap talk equilibria in which only the High-type is not truthful, while the Low-type is truthful. We analyse this particular type of partial revelation because in the fully revealing equilibrium, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of Low-type. We identify the unique partially revealing cheap talk equilibrium with the type-coordination property in this set of equilibria[5] and prove its existence based on the prior probability of the High-type being within a range that turns out to be non-overlapping and higher than that for the fully revealing equilibrium (Theorem 2). We show that these cheap talk equilibria are more efficient than the Bayesian-Nash equilibrium of the game without any communication and illustrate all these results using numerical examples.

We also consider the scenario when only one of the players is allowed to talk in our game, to understand the difference between one-sided and two-sided cheap talk. We find (Theorem 3) that truthful cheap talk by one player only is not possible in any meaningful equilibrium.

Finally, we also consider non-canonical message spaces at the cheap-talk stage; we identify a new equilibrium with a bigger message space (including the types) and find (Theorem 4) that, with the help of more messages, truthfulness and the desirable coordination together may be achieved even when the direct truthful cheap talk equilibrium does not exist.

2 MODEL

2.1 The Game

We consider a version of the BoS with incomplete information as given below, in which each of the two players has a set of strategies S_i containing two pure strategies, namely, A and B, i.e., $S_i = \{A, B\}$, $i = 1, 2$. Let $S = S_1 \times S_2$ denote the set of strategy combinations of the two players. The payoffs are as in the following table, in which the value of $t_i \in T_i$ is the private information of player i, $i = 1, 2$,

identical symmetric situation ex-ante, we study (type and player) symmetric cheap talk equilibria, following the tradition in the literature (as in Farrell, 1987 and Banks and Calvert, 1992).

[5] We also characterise the complete set of partially revealing cheap talk equilibria in which only the High-type is not truthful while the Low-type is truthful (Proposition 2 in this paper).

with $0 < t_i < 1$. We assume that t_i is a discrete random variable that takes only two values L and H, where, $0 < L < H < 1$, whose realisation is only observed by player i. So, $T_i = \{L, H\}$, $i = 1, 2$. For $i = 1, 2$, we henceforth refer to the values of t_i as player i's type (Low, High). We further assume that each player's type is independently drawn from the set $\{L, H\}$ according to a probability distribution with $Prob(t_i = H) = p(H) = p \in [0, 1]$. Also, the payoffs to both players from the miscoordinated outcome is normalised to 0, while the payoff to player 1 (player 2) from (A, A) $((B, B))$ is normalised to 1.

		Player 2: A	Player 2: B
Player 1	A	$1, t_2$	$0, 0$
	B	$0, 0$	$t_1, 1$

These payoffs will also formally be denoted by the players' utility functions $u_i : S \times T_i \to \mathbb{R}, i = 1, 2$. Note that player i's utility depends here on own type t_i only and not on the other player's private information t_j.

The unique symmetric Bayesian-Nash equilibrium[6] of this game[7] can be characterised by $\sigma_i(s_i | t_i)$, the probability that player i of type t_i plays the pure strategy s_i.

Proposition 1 *The unique symmetric Bayesian Nash equilibrium of the BoS with incomplete information is given by the following strategy for player* 1 *(player* 2*'s strategy is symmetric and is given by* $\sigma_1(A | t) = \sigma_2(B | t)$, $t = H, L$*):*

$\sigma_1(A | H) = 0$ *and* $\sigma_1(A | L) = \frac{1}{(1-p)(1+L)}$ *when* $p < \frac{L}{1+L}$,

$\sigma_1(A | H) = 0$ *and* $\sigma_1(A | L) = 1$ *when* $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$,

$\sigma_1(A | H) = 1 - \frac{H}{p(1+H)}$ *and* $\sigma_1(A | L) = 1$ *when* $p > \frac{H}{1+H}$.

The proof is straightforward and hence has been omitted here.

2.2 Cheap Talk

We study an extended game in which the players are first allowed to have a round of simultaneous canonical cheap talk intending to reveal their private information before they play the above BoS. In the first (cheap talk) stage of this extended game, each player i simultaneously chooses a costless and nonbinding announcement τ_i from the set $\mathcal{T}_i = \{L, H\}$. Then, given a pair of announcements (τ_1, τ_2),

[6] The corresponding game with complete information with commonly known values t_1 and t_2, has two pure Nash equilibria, (A, A) and (B, B), and a mixed Nash equilibrium in which player 1 plays A with probability $\frac{1}{1+t_2}$ and player 2 plays B with probability $\frac{1}{1+t_1}$.

[7] In a similar game, Banks and Calvert (1992) also provided a similar characterisation.

in the second (action) stage of this extended game, each player i simultaneously chooses an action s_i from the set S_i.

An announcement strategy in the first stage for player i is a function $a_i : T_i \to \Delta(\mathcal{T}_i)$, where $\Delta(\mathcal{T}_i)$ is the set of probability distributions over $\mathcal{T}_i$. We write $a_i(H\,|t_i\,)$ for the probability that strategy $a_i(t_i)$ of player i with type t_i assigns to the announcement H. Thus, the announcement τ_i of player i with type t_i is a random variable drawn from $\mathcal{T}_i$ according to the probability distribution with $Prob(\tau_i = H) = a_i(H\,|t_i\,)$.

Beliefs for player i are given by $\widetilde{p_i} : \mathcal{T}_j \to \Delta(T_i), i, j = 1, 2$. We will denote i's posterior belief by $\widetilde{p_i}(H\,|\tau_j\,) = Prob(t_j = H\,|\tau_j\,)$.

In the second (action) stage, a strategy for player i is a function $\sigma_i : T_i \times \mathcal{T}_1 \times \mathcal{T}_2 \to \Delta(S_i)$, where $\Delta(S_i)$ is the set of probability distributions over S_i . We write $\sigma_i(A\,|t_i;\tau_1,\tau_2\,)$ for the probability that strategy $\sigma_i(t_i;\tau_1,\tau_2)$ of player i with type t_i assigns to the action A when the first stage announcements are (τ_1,τ_2). Thus, player i with type t_i's action choice s_i is a random variable drawn from $\{A, B\}$ according to a probability distribution with $Prob(s_i = A) = \sigma_i(A\,|t_i;\tau_1,\tau_2\,)$. Given a pair of realised action choices $(s_1, s_2) \in S_1 \times S_2$, the corresponding outcome is generated. Thus, given a strategy profile $((a_1,\sigma_1),(a_2,\sigma_2))$, one can find the players' actual payoffs from the induced outcomes in the type-specific payoff matrix of the BoS and hence, the (ex-ante) expected payoffs. As the game is symmetric, in our analysis, we maintain the following notion of symmetry in the strategies, for the rest of the paper.

Definition 1 *A strategy profile $((a_1,\sigma_1),(a_2,\sigma_2))$ is called announcement-symmetric (in the announcement stage) if $a_i(H\,|t_i\,) = a_{-i}(H\,|t_i\,)$; a strategy profile is called action-symmetric (in the action stage) if $\sigma_i(A\,|t;\tau_1,\tau_2\,) = \sigma_{-i}(B\,|t;\tau_2,\tau_1\,)$, for all t,τ_1,τ_2. A strategy profile is called symmetric if it is both announcement-symmetric and action-symmetric.*

Note that Definition 1 preserves symmetry for both players and the types for each player. We consider the following standard notion of Perfect Bayesian Equilibrium (PBE)[8] in this two-stage cheap talk game.

Definition 2 *A symmetric strategy profile $((a_1,\sigma_1),(a_2,\sigma_2))$ together with beliefs $(\widetilde{p_1},\widetilde{p_2})$ is called a symmetric cheap talk equilibrium if it is a Perfect Bayesian Equilibrium (PBE) of the game with cheap talk, i.e., each player is playing optimally at all his information sets given the strategy of the other player and the beliefs are updated according to the Bayes rule whenever possible.*

Formally, $((a_1,\sigma_1),(a_2,\sigma_2))$ and $(\widetilde{p_1},\widetilde{p_2})$ is a PBE of the game with cheap talk if

[8] Our definition follows the standard formulation of PBE in the literature (see for example, Fudenberg and Tirole, 1991 and Baliga and Morris, 2002).

(1) $\forall t_i, \forall i \neq j, i, j = 1, 2$

$a_i(\tau_i \left| t_i \right.) > 0 \Longrightarrow$

$\tau_i \in \arg\max_{\tau_i' \in T_i} \sum_{t_j} p\left(t_j\right) \sum_{\tau_j} a_j(\tau_j \left| t_j \right.) \sum_{s \in S} \left[\sigma_i(s_i \left| t_i; \tau_i', \tau_j \right.) \sigma_j(s_j \left| t_j; \tau_i', \tau_j \right.)\right] u_i\left(s, t_i\right)$

(2) $\forall t_i, \tau_i, \tau_j, \forall i \neq j, i, j = 1, 2$

$\sigma_i(s_i \left| t_i; \tau_i, \tau_j \right.) > 0 \Longrightarrow$

$s_i \in \arg\max_{s_i' \in \mathcal{S}_i} \sum \widetilde{p_i}(t_j \left| \tau_j \right.) \sum_{s_j \in S_j} \sigma_j(s_j \left| t_j; \tau_i', \tau_j \right.) u_i\left(s_i', s_j, t_i\right)$

(3) $\forall i \neq j, i, j = 1, 2$

$\widetilde{p_i}(t_j \left| \tau_j \right.) = \frac{a_j(\tau_j | t_j) p(t_j)}{\sum_{t_j' \in T_j} a_j(\tau_j \left| t_j' \right.) p\left(t_j'\right)}$ *if* $\sum_{t_j' \in T_j} a_j(\tau_j \left| t_j' \right.) p\left(t_j'\right) > 0$

and $\widetilde{p_i}(. \left| \tau_j \right.)$ *is any probability distribution on* T_i *if* $\sum_{t_j' \in T_j} a_j(\tau_j \left| t_j' \right.) p\left(t_j'\right) = 0.$

In Definition 2 above, Condition *(1)* ensures optimality at the cheap talk stage. For example, if the announcement strategy $a_i(. \left| t_i \right.)$ is completely mixed, then condition *(1)* implies that both the messages ($\tau_i = H$ and $\tau_i = L$) should provide the same expected payoff to player i with type t_i. If $a_i(. \left| t_i \right.)$ is a pure strategy, then condition *(1)* will yield a weak inequality whereby the expected payoff from the chosen pure strategy announcement and then following the equilibrium strategy at the action stage is at least as high as the expected payoff from choosing the other message and subsequently using the optimal strategy at the action stage (which could possibly be a deviation from the prescribed equilibrium profile). Similarly, Condition *(2)* ensures optimality at the action stage whereby a completely mixed action strategy yields an equality constraint for expected payoffs (using suitable posterior beliefs) and a pure strategy yields an inequality constraint. Finally, Condition *(3)* ensures that posterior beliefs are derived using Bayes rule.

Definition 2 suggests that a symmetric cheap talk equilibrium can be characterised by a set of (symmetric) equilibrium constraints (2 for the announcement stage and another possible 8 for the action stage).

3 MAIN RESULTS

The main purpose of our paper is to find, if exists, an equilibrium with truthful talk. We thus first consider the possibility of full revelation of the types as a result of our canonical cheap talk. Subsequently, we present and analyse some other cheap talk equilibria in this section.

3.1 Fully Revealing Equilibrium

We consider a specific class of strategies in this subsection where we impose the property that the cheap talk announcement should be fully revealing.

Definition 3 *A symmetric strategy profile* $((a_1, \sigma_1), (a_2, \sigma_2))$ *is called fully revealing if the announcement strategy* a_i *reveals the true types with certainty, i.e.,* $a_i(H\,|H\,) = 1$ *and* $a_i(H\,|L\,) = 0$.

We now characterise the fully revealing symmetric cheap talk equilibrium. We first consider a specific fully revealing (separating) strategy profile that we call $S_{separating}$, influenced by the equilibrium action profile in Farrell (1987) for the complete information version of this game. In this strategy profile, the players announce their types truthfully and then in the action stage, they play the mixed Nash equilibrium strategies of the complete information BoS when both players' types are identical and they play (B, B) $((A, A))$, when only player 1's type is H (L).

We state our first result below.[9]

Theorem 1 $S_{separating}$ *is the unique fully revealing symmetric cheap talk equilibrium and it exists only for* $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p \leq \frac{LH+LH^2}{1+L+LH+LH^2}$.

Before proving Theorem 1, we first observe the following fact that follows from Definitions 2 and 3.

In a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, the players' strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is a (pure or mixed) Nash equilibrium of the BoS with values t_1 and t_2, $\forall t_1, t_2 \in \{H, L\}$. Thus, in a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, conditional on the announcement profile (H, H) or (L, L), the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information BoS, that is, whenever $t_1 = t_2$, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is the mixed Nash equilibrium of the BoS with values $t_1 = t_2$.

Based on the above fact, one can easily identify all the candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric. It implies that these profiles are differentiated only by the actions played when $t_1 \neq t_2$, that is, when the players' types are (H, L) and (L, H).

The proof of Theorem 1 may now be completed easily; we have postponed the details of the rest of the proof to the Appendix of this paper.

The following couple of claims illustrate some features of the equilibrium $S_{separating}$. The claims are easy to establish and hence we have omitted the formal proofs for them.

Claim 1 *The ex-ante expected payoff for any player from* $S_{separating}$ *is given by* $EU_{separating} = p^2\frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2\frac{L}{1+L}$, *which is increasing over the range of* p *where it exists.*

Claim 2 *The upper bound for* p *in Theorem 1,* $\frac{LH+LH^2}{1+L+LH+LH^2}$ *is always* $< \frac{1}{2}$, *since* $\frac{1}{2} - \frac{HL+H^2L}{1+L+HL+H^2L} = \frac{(1+L-LH-LH^2)}{2(1+L+HL+H^2L)} > 0$, *as long as* $L < H < 1$.

[9] Theorem 1 in this paper corrects and thus improves upon the main result presented in Ganguly and Ray (2009).

To understand why p must lie in such a low range for this equilibrium to exist (as stated in Theorem 1 and Claim 2), consider the incentives for deviations by player 1 at the announcement stage. By deviating and claiming to be an L-type, player 1(H-type) gains $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a H-type (with probability p) and loses[10] $H - \frac{H}{1+L} = \frac{HL}{1+L}$ when player 2 is a L-type (with probability $1-p$). Since $\frac{1}{1+H} - \frac{HL}{1+L} = \frac{(1+L-LH-LH^2)}{(1+H)(1+L)} > 0$, the gain from the deviation when playing against player 2(H-type) is bigger than the loss when playing against player 2(L-type). If a H-type is equally or more likely than a L-type, then player 1(H-type) will obviously deviate and truthful revelation will not be an equilibrium. So, p must be $< \frac{1}{2}$. Indeed, p needs to be small enough to make the above deviation unattractive and the precise value of p for which this holds is $\frac{HL+H^2L}{1+L+HL+H^2L}$ or less. However, p cannot be too close to 0 either. This is because of incentives for deviations by player 1(L-type). By deviating and claiming to be an H-type, player 1(L-type) gains $L - \frac{L}{1+L} = \frac{L^2}{1+L}$ when player 2 is a L-type (with probability $1-p$) and loses $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a H-type (with probability p). Since $\frac{1}{1+H} - \frac{L^2}{1+L} = \frac{(1+L-HL^2-L^2)}{(1+H)(1+L)} > 0$, the loss from the deviation when playing against player 2(H-type) is bigger than the gain when playing against player 2(L-type). The expected gain will outweigh the expected loss only if a L-type is much more likely than a H-type (and L is bigger than 0). Hence, player 1(L-type) would deviate at the cheap talk stage only if p is too close to 0.

3.2 Partially Revealing Equilibrium

The fully revealing symmetric cheap talk equilibrium exists only for a moderately low range of the prior probability p. One may now ask what sort of equilibria, if any, exists for any given p outside this range. Understandably, it is not easy to characterise all possible equilibria for this game.

In this subsection, we aim to characterise a specific class of partially revealing symmetric equilibria of the above cheap talk game. As noted earlier, in the fully revealing equilibrium, the H-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of L-type. We thus find it natural to analyse the type of partial revelation in which only the L-type truthfully reveals while the H-type does not.

Formally, we consider a symmetric announcement strategy profile in which the H-type of player i announces H with probability r and L with probability $(1-r)$ and the L-type of player i announces L with probability 1, i.e., $a_i(H\,|H) = r$ and $a_i(H\,|L) = 0$. Clearly, after the cheap talk phase, the possible message profiles (τ_1, τ_2) that the H-type of player 1 may receive are (H, H), (H, L), (L, H) or (L, L) while the L-type of player 1 may receive either (L, H) or (L, L).

Let us denote an action-strategy of player 1 by $\sigma_1(A\,|H; H, H) = q_0$, $\sigma_1(A\,|H; H, L) = q_1$, $\sigma_1(A\,|H; L, H) = q_2$, $\sigma_1(A\,|H; L, L) = q_3$, $\sigma_1(A\,|L; L, H) = q_4$ and $\sigma_1(A\,|L; L, L) = q_5$. By sym-

[10] Note that after deviating in the cheap talk stage, player 1 (H-type) may deviate at the action stage as well. In fact, the optimal deviation strategy for player 1 (H-type) is to play action B when player 2 is a L-type.

metry, a partially revealing symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ in our set-up can thus be identified by $(r, q_0, q_1, q_2, q_3, q_4, q_5)$.

First note that, on receiving the message profile (H, H), the players know the true types and hence in any such partially revealing symmetric cheap talk equilibrium, q_0 has to correspond to the mixed Nash equilibrium of the complete information BoS with values H and H. Thus, $q_0 = \frac{1}{1+H}$.

One may indeed characterise the whole set of partially revealing symmetric cheap talk equilibria in this set up (in which only the L-type is truthful), by characterising the equilibrium values of $(r, q_1, q_2, q_3, q_4, q_5)$, using the following equilibrium conditions.

If q_1, q_2, q_3, q_4 and q_5 correspond to completely mixed strategies in the action stage, then we must have the following five conditions for player 1 to be indifferent between playing A and B (where the LHS in each equation is the expected payoff from A and the RHS in each equation is the expected payoff from B):

Player 1(H-type) receiving the message profile (L, H):

$$1 - q_1 = q_1 H, \tag{1}$$

Player 1(H-type) receiving the message profile (H, L):

$$(\frac{p - rp}{1 - rp})(1 - q_2) + (1 - \frac{p - rp}{1 - rp})(1 - q_4) = (\frac{p - rp}{1 - rp})q_2 H + (1 - \frac{p - rp}{1 - rp})q_4 H, \tag{2}$$

Player 1(H-type) receiving the message profile (L, L):

$$(\frac{p - rp}{1 - rp})(1 - q_3) + (1 - \frac{p - rp}{1 - rp})(1 - q_5) = (\frac{p - rp}{1 - rp})q_3 H + (1 - \frac{p - rp}{1 - rp})q_5 H, \tag{3}$$

Player 1(L-type) receiving the message profile (L, L):

$$(\frac{p - rp}{1 - rp})(1 - q_3) + (1 - \frac{p - rp}{1 - rp})(1 - q_5) = (\frac{p - rp}{1 - rp})q_3 L + (1 - \frac{p - rp}{1 - rp})q_5 L, \tag{4}$$

Player 1(L-type) receiving the message profile (L, H):

$$1 - q_1 = q_1 L. \tag{5}$$

Also, in the cheap talk phase, player 1(H-type), who is using a completely mixed strategy, should be indifferent between announcing H and L, which implies

$$\begin{aligned} & (1-p)(q_1(1-q_4) + (1-q_1)q_4 H) + p(r\frac{H}{1+H} + (1-r)(q_1(1-q_2) + (1-q_1)q_2 H)) \\ = \; & p(r(q_2(1-q_1) + (1-q_2)q_1 H) + (1-r)(q_3(1-q_3) + (1-q_3)q_3 H)) \\ & + (1-p)(q_3(1-q_5) + (1-q_3)q_5 H). \end{aligned} \tag{6}$$

where the LHS of (6) is the expected payoff from announcing H and the RHS is the expected payoff from announcing L.

Finally, in the cheap talk phase, it should be incentive compatible for player 1(L-type) to announce L, which implies

$$\begin{aligned} & (1-p)\left(q_5\left(1-q_5\right)(1+L)\right)+p\left(r\left(q_4\left(1-q_1\right)+(1-q_4)q_1L\right)+(1-r)\left(q_5\left(1-q_3\right)+(1-q_5)q_3L\right)\right) \\ \geq \;& \underset{x}{Max}\,(1-p)\left(x\left(1-q_4\right)+(1-x)\,q_4L\right)+p(r\frac{H}{1+H}+(1-r)\left(x\left(1-q_2\right)+(1-x)\,q_2L\right)) \end{aligned} \quad (7)$$

The LHS of (7) is the expected payoff from announcing L and following the equilibrium strategy thereafter while the RHS is the expected payoff from deviating and announcing H and then choosing the optimal strategy in the action phase given the deviation in the cheap talk phase. So, x is the optimal probability of playing A in the action phase after player 1(L-type) deviates and announces H and receives the message profile (H, L). Note that the RHS of this last inequality constraint (7) allows player 1(L-type) to deviate in both stages of the game and hence, (7) checks the player's incentives against the best possible deviation.

By virtue of symmetry, the equilibrium conditions for player 2 are identical to the above. Using these equilibrium conditions, one can prove that certain profiles as listed in the proposition below constitute this equilibrium set.

Proposition 2 *The following profiles are the only partially revealing symmetric cheap talk equilibria in which only the L-type is truthful:*

(i) $q_1 = \frac{1}{1+H}$, $q_2 = q_3 = \frac{p+Hp-rp-H}{p+Hp-rp-Hrp}$, $q_4 = q_5 = 1$ *with any* $0 < r \leq 1 - \frac{H(1-p)}{p}$*; exists when* $p > \frac{H}{1+H}$,

(ii) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = \frac{1}{1+L+LH+LH^2-p-Lp-LHp-LH^2p}$ *and* $r = \frac{LH+LH^2}{p+Lp+LHp+LH^2p}$*; exists when* $\frac{LH+LH^2}{1+L+LH+LH^2} < p < \frac{L+LH+LH^2}{1+L+LH+LH^2}$,

(iii) $q_1 = 0$, $q_2 = 1$, $q_3 = \frac{p+Hp+H^2p+H^3p-H-H^2-H^3}{p+Hp+H^2p+H^3p-H^2-H^3}$, $q_4 = q_5 = 1$ *and* $r = \frac{H^2}{p+H^2p}$*; exists when* $p > \frac{H+H^2+H^3}{1+H+H^2+H^3}$,

(iv) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = 1$ *and* $r = \frac{H+H^2}{1+H+H^2}$*; exists when* $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$,

(v) $q_1 = \frac{1}{1+H}$, $q_2 = q_3 = 0$, $q_4 = q_5 = 1$ *and* $r = \frac{p+Hp-H}{p}$*; exists when* $\frac{H}{1+H} < p < 1$.

We are not presenting the details of the proof of Proposition 2 which can be found in a previous discussion paper version of our work (Ganguly and Ray, 2013).

3.3 Coordination

One may note that $S_{separating}$ features a specific form of coordination in which the players play (B, B) ((A, A)) when only player 1's type is H (L), that is, when the players' types are different, players fully coordinate on a pure Nash equilibrium outcome that generate the ex-post efficient payoffs of 1 and H. We call this property "type-coordination".

Definition 4 *A strategy profile is said to have the type-coordination property if the induced outcome is (A, A) and (B, B), when the players' true type profile is (L, H) and (H, L), respectively.*

Clearly, the type-coordination property can be achieved in other kinds of equilibria. Indeed, the Bayesian Nash equilibrium of the game without the cheap talk (as mentioned in Proposition 1) also satisfies type-coordination property when the prior p is between $\frac{L}{1+L}$ and $\frac{H}{1+H}$ (for example, between $\frac{1}{4}$ and $\frac{2}{5}$ for the parameters $L = \frac{1}{3}$, $H = \frac{2}{3}$).

Although the type-coordination property can be obtained in the fully revealing cheap talk equilibrium or in the Bayesian Nash equilibrium, one might still ask whether it is possible to obtain type-coordination where players do not reveal their types truthfully. This motivates us to check whether at all type-coordination can be obtained in any of the partially revealing equilibria, as described in Proposition 2.

In a partially revealing equilibrium in which only the L-type truthfully reveals while the H-type does not, following Proposition 2, note that for the type-coordination property to hold, we need profiles satisfying $q_1 = 0$, $q_3 = 0$ and $q_5 = 1$. Using symmetry, for player 2(H-type), we then must have $\sigma_2(A\,|H; L, H\,) = 1 - q_1 = 1$. This implies that in any such profile, $q_2 = 1$ and $q_4 = 1$. Thus, a candidate partially revealing equilibrium profile with the type-coordination property must have $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

We now state our second main result. The proof has been postponed to the Appendix.

Theorem 2 *In a partially revealing symmetric cheap talk equilibrium, in which only the L-type is truthful, that satisfies the type-coordination property, r must be $\frac{H+H^2}{1+H+H^2}$; this equilibrium exists only when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$.*

Note that the profile in Theorem 2 above is the same as that given in *(iv)* in Proposition 2. Let the equilibrium profile stated in Theorem 2 be called $S_{pooling}$.

To understand why p must lie within such a range for $S_{pooling}$ to be an equilibrium (as stated in Theorem 2), consider the incentives for deviations by player 1 at the action stage. According to the above strategy profile, on receiving the message profile (L, L), player 1(H-type) needs to play B. Given that player 2(H-type) plays A and player 2(L-type) plays B, player 1(H-type) will indeed play B only if he believes that player 2 is more likely to be an L-type than an H-type. This means that player 1(H-type)'s posterior belief about player 2 being an H-type should not be too high. If we denote this posterior belief by $p\prime$, then $p\prime = P(t_i = H\,|\tau_i = L\,) = \frac{p-rp}{1-rp}$. Since this posterior $p\prime$ is an increasing function of the prior p ($\frac{\partial}{\partial p}(\frac{p-rp}{1-rp}) = \frac{(1-r)}{(1-rp)^2} > 0$), the constraint that $p\prime$ should not be too high implies that the prior p cannot be very high either.[11] Hence, there is an upper bound for p that is strictly less

[11] If p were to be equal to 1, i.e., player 2 were certainly an H-type, player 1(H-type) would then definitely have preferred playing A, not B.

than 1. Similarly, according to the above strategy profile, after receiving the message profile (L, L), player 1(L-type) needs to play A. Again, given that player 2(H-type) plays A and player 2(L-type) plays B, player 1(L-type) will play A only if the posterior $p\prime$ is not too small which explains the lower bound on p.

The following couple of claims illustrate some features of the equilibrium $S_{pooling}$. These two claims are easy to establish and hence we have omitted the formal proofs for them.

Claim 3 *The ex-ante expected payoff for any player from the equilibrium $S_{pooling}$ is given by* $EU_{pooling} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$.

Claim 4 *The upper bound of the range for p in Theorem 2 does not involve L, is increasing in H and is bounded by $\frac{3}{4}$.*

Our cheap talk equilibria, $S_{separating}$ and $S_{pooling}$ both satisfy the type-coordination property. We end this subsection with the following observation.

Claim 5 *For a fixed value of H and L, the lower bound for p for $S_{pooling}$ to exist is bigger than the upper bound for p for $S_{separating}$ to exist.*

Thus, these two different equilibria with the type-coordination property exist for distinct values of p. For $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$, when $S_{pooling}$ exists as an equilibrium, $S_{separating}$ is not an equilibrium because the H-type does not want to truthfully reveal his information. Allowing the H-type to reveal his information partially in the cheap talk stage helps sustain the partially revealing equilibrium.

3.4 Numerical Illustrations

We now illustrate our main results by a specific numerical example. Let us consider the following version of our game in which the payoff of the H-type of player 1 from (B, B) is $\frac{2}{3}$, twice that of the L-type as described in the following table.

	A	B
A	$1, \frac{2}{3}$	$0, 0$
B	$0, 0$	$\frac{2}{3}, 1$

Types: HH

	A	B
A	$1, \frac{1}{3}$	$0, 0$
B	$0, 0$	$\frac{2}{3}, 1$

Types: HL

	A	B
A	$1, \frac{2}{3}$	$0, 0$
B	$0, 0$	$\frac{1}{3}, 1$

Types: LH

	A	B
A	$1, \frac{1}{3}$	$0, 0$
B	$0, 0$	$\frac{1}{3}, 1$

Types: LL

Also, take the (independent) prior probability of the H-type, $Prob(t_i = \frac{2}{3})$ to be $\frac{1}{5}$. Then, the unique symmetric Bayesian Nash equilibrium of this specific Bayesian game is given by the following symmetric strategy profile: player 1 plays A with probability $\frac{15}{16}$ when the type is Low and plays the pure strategy B when the type is High (player 2's strategy is symmetric and is B with probability $\frac{15}{16}$ when the type is Low and A when the type is High) which generates the following distribution over the outcomes for different type profiles (states of the world).

Types: HH

	A	B
A	0	0
B	1	0

Types: HL

	A	B
A	0	0
B	$\frac{1}{16}$	$\frac{15}{16}$

Types: LH

	A	B
A	$\frac{15}{16}$	0
B	$\frac{1}{16}$	0

Types: LL

	A	B
A	$\frac{15}{256}$	$\frac{225}{256}$
B	$\frac{1}{256}$	$\frac{15}{256}$

From the above distribution over outcomes, one may observe that the unique Bayesian Nash equilibrium for this specific game involves fair amount of miscoordination.

From Theorem 1, for these parameter values ($L = \frac{1}{3}$ and $H = \frac{2}{3}$), we know that the range of the prior p for which $S_{separating}$ exists is $\frac{5}{41} (\simeq 0.12) \leq p \leq \frac{5}{23} (\simeq 0.22)$. For example, when $p = \frac{5}{23}$, one may check that the payoff from $S_{separating}$ is $\frac{241}{529} (\simeq 0.46)$ and that $S_{separating}$ generates the following distribution over the outcomes.

Types: HH

	A	B
A	$\frac{6}{25}$	$\frac{9}{25}$
B	$\frac{4}{25}$	$\frac{6}{25}$

Types: HL

	A	B
A	0	0
B	0	1

Types: LH

	A	B
A	1	0
B	0	0

Types: LL

	A	B
A	$\frac{3}{16}$	$\frac{9}{16}$
B	$\frac{1}{16}$	$\frac{3}{16}$

Following Theorem 2, $S_{pooling}$ for these parameter values exists for p between $\frac{19}{46}$ ($\simeq 0.41$) and $\frac{38}{65}$ ($\simeq 0.58$); in $S_{pooling}$, the L-type is truthful but the H-type partially reveals his true type with probability $\frac{10}{19}$ ($\simeq 0.53$). When $p = \frac{38}{65}$, the payoff from such an equilibrium also turns out to be $\frac{38}{65} (\simeq 0.58)$. The corresponding equilibrium distribution over the outcomes is as follows.

Types: HH

	A	B
A	$\frac{141}{361}$	$\frac{36}{361}$
B	$\frac{97}{361}$	$\frac{141}{361}$

Types: HL

	A	B
A	0	0
B	0	1

Types: LH

	A	B
A	1	0
B	0	0

Types: LL

	A	B
A	0	1
B	0	0

Note that although the above equilibrium achieves type-coordination, in this case, there is a complete miscoordination in the LL state.

3.5 Cheap Talk vs. Bayesian Nash Equilibria

As noted earlier, it is possible to achieve type-coordination in the unique symmetric Bayesian Nash equilibrium (BNE) of the BoS (without the cheap talk stage) itself when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$ (see Proposition 1).

It is also conceivable that for some parameter values of L and H (such as, $L = 0.2$, $H = 0.9$), the ranges of p where $S_{separating}$ and $S_{pooling}$ respectively exist, do separately overlap with the interval $[\frac{L}{1+L}, \frac{H}{1+H}]$. Hence, there exist possible values of p for which both $S_{separating}$ and the BNE achieve type-coordination (such as, $p = 0.2$ for the parameter values $L = 0.2$, $H = 0.9$) and similarly, values of p for which both $S_{pooling}$ and the BNE achieve type-coordination (such as, $p = 0.4$ for the parameter values $L = 0.2$, $H = 0.9$).

This raises the obvious question of how these different equilibria compare in terms of expected utilities for the players. Do the equilibria from the game with cheap talk, i.e., $S_{separating}$ and $S_{pooling}$, always outperform the BNE without cheap talk when they simultaneously exist? The answer is yes!

First, we note that the range of p where $S_{separating}$ exists, can only overlap with the low and middle ranges of p in the description of the BNE (as in Proposition 1), i.e., the intervals $[0, \frac{L}{1+L})$ and $[\frac{L}{1+L}, \frac{H}{1+H}]$. This is because $\frac{H}{1+H} - \frac{LH+LH^2}{1+L+LH+LH^2} = \frac{H(1-HL)}{(1+H)(1+L+LH+LH^2)} > 0$ and $\frac{L}{1+L} - \frac{L^2+L^2H}{1+L+L^2+L^2H} = \frac{L(1-HL)}{(1+L)(1+L+L^2+L^2H)} > 0$. Next, we note that the range of p where $S_{pooling}$ exists, can only overlap with the middle and high ranges of p in the description of the BNE (as in Proposition 1), i.e., the intervals $[\frac{L}{1+L}, \frac{H}{1+H}]$ and $(\frac{H}{1+H}, 1]$. This is because $\frac{L+LH+LH^2}{1+L+LH+LH^2} - \frac{L}{1+L} = \frac{HL(1+H)}{(1+L)(1+L+LH+LH^2)} > 0$ and $\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{H}{1+H} = \frac{H^2}{1+H+H^2+H^3} > 0$. We now state the following result comparing these different equilibria.

Proposition 3 *The expected payoff for each player from $S_{separating}$ ($S_{pooling}$) is greater than that from the Bayesian Nash equilibrium of the game without cheap talk when the Bayesian Nash equilibrium and $S_{separating}$ ($S_{pooling}$) simultaneously exist.*

Proposition 3 thus suggests that it is better to talk! The proof is in the Appendix.

3.6 Cheap Talk vs. Mediated Equilibria

Banks and Calvert (1992) characterised the (ex-ante) efficient symmetric incentive compatible direct mechanism for a similar game so that the players are truthful and obedient to the mechanism (mediator). Following Banks and Calvert (1992), one may analyse (as in Ganguly and Ray 2009) a (direct) *symmetric mediated equilibrium* that provides the players with incentives (i) to truthfully reveal their types to the mediator and (ii) to follow the mediator's recommendations following their type-announcements. Clearly, our cheap talk equilibria can be achieved as outcomes of such mediated equilibria using incentive compatible mechanisms. Formally, one can easily prove that (i) the distribution over the outcomes generated by $S_{separating}$ can be achieved as a symmetric mediated equilibrium if $\frac{L^2+2L^2H+L^2H^2}{1+L+H+LH^2+L^2+L^2H+L^2H^2+H^2} \leq p \leq \frac{H-L+LH^2+L^2H+L^2H^2+H^2}{1+L+H+LH^2+L^2+L^2H+L^2H^2+H^2}$ and (ii) the distribution over the outcomes generated by $S_{pooling}$ can be achieved as a symmetric mediated equilibrium if $\frac{L+LH+LH^2}{1+L+LH^2+H^2} \leq p \leq \frac{H+H^2+H^3}{1+H+H^2+H^3}$. Not surprisingly, these ranges of p strictly contain the corresponding ranges for the cheap talk equilibria implying a larger range of p for which the corresponding mechanism is in equilibrium. Rather intuitively, this indicates that there are priors for which an outcome can be obtained as an equilibrium via a direct mechanism but not using the unmediated one-round cheap talk that only allows direct communication between players of different types.

4 EXTENSIONS

We present a couple of further issues related to our main results below.

4.1 One-sided Talk

One-sided cheap talk with two-sided private information has also been studied in the literature (see, for example, Seidmann (1990) and more recently, Moreno de Barreda (2012)). One thus may be interested to know whether the properties of truthfulness and type-coordination of the two-sided cheap talk equilibria can be achieved with one-sided cheap talk in our game, when only one player (say, player 1) talks.

To do so, we assume that player 1 chooses a costless and nonbinding announcement τ_1 from the set $\mathcal{T}_1 = \{L, H\}$.We now write $\sigma_i(A\,|t_i;\tau_1)$ for the probability that strategy $\sigma_i(t_i;\tau_1)$ of player i with type t_i assigns to the action A when the first stage announcement by player 1 is τ_1.

We first consider below two specific strategy profiles which we believe are closest to the two equilibrium strategy profiles studied earlier with two-sided cheap talk and we show that these strategy profiles are no longer equilibrium profiles.

The first strategy profile we analyse concerns the situation where player 1 reveals his information truthfully. Consider the following strategy profile: in the cheap talk stage, player 1 reports his type truthfully, i.e., $a_1(H\,|H) = 1$ and $a_1(H\,|L) = 0$; in the action stage, player 1's strategy consists of any $0 \leq \sigma_1(A\,|H;H) \leq 1$ and $0 \leq \sigma_1(A\,|L;L) \leq 1$ and player 2's strategy is given by $\sigma_2(A\,|H;H) = \frac{H}{1+H}$, $\sigma_2(A\,|H;L) = 1$, $\sigma_2(A\,|L;H) = 0$ and $\sigma_2(A\,|L;L) = \frac{L}{1+L}$. Call this strategy $S^{onesided}_{separating}$. It is easy to prove that $S^{onesided}_{separating}$ is not an equilibrium in the game with one-sided cheap talk where only player 1 talks. To see this note that, in the action stage, player 1(H-type)'s expected payoff from playing A is $p\frac{H}{1+H}$ whereas his expected payoff from playing B is $p\frac{H}{1+H} + (1-p)H$; hence, $\sigma_1(A\,|H;H) = 0$. But this implies that player 2(H-type) should play the pure strategy B after player 1(H-type) talks, i.e., $\sigma_2(A\,|H;H\,) = 0$; therefore, $S^{onesided}_{separating}$ cannot be an equilibrium.

The second strategy profile concerns the situation where player 1(H-type) partially reveals his information, while player 1(L-type) announces L truthfully. Formally, consider the following strategy profile: in the cheap talk stage, player 1 reveals his type partially, i.e., $a_1(H\,|H\,) = r$ and $a_1(H\,|L\,) = 0$; in the action stage, player 1's strategy consists of any $0 \leq \sigma_1(A\,|H;H\,) \leq 1$, $0 \leq \sigma_1(A\,|H;L\,) \leq 1$ and $0 \leq \sigma_1(A\,|L;L\,) \leq 1$ and player 2's strategy is given by $\sigma_2(A\,|H;H\,) = \frac{H}{1+H}$, $\sigma_2(A\,|H;L\,) = 1$, $\sigma_2(A\,|L;H\,) = 0$, $\sigma_2(A\,|L;L\,) = \frac{L}{1+L}$. Call this strategy $S^{onesided}_{pooling}$. Following the same logic as in the case of $S^{onesided}_{separating}$, one can also show that $S^{onesided}_{pooling}$ is not an equilibrium.

We are now going to show a more general result, namely, that any strategy profile involving truthful revelation in the cheap talk stage does not lead to any meaningful equilibrium. To show this, we thus focus our attention only on equilibria where at least some of the actions in the second stage depend on the announcement from the first stage in a non-trivial manner.

Definition 5 *A strategy profile in the game with one-sided cheap talk (by player* 1*) is called responsive if at least one of the following holds:*

(i) $\sigma_1(A\,|H;H\,) \neq \sigma_1(A\,|L;L\,)$

(ii) $\sigma_2(A\,|H;H\,) \neq \sigma_2(A\,|H;L\,)$

(iii) $\sigma_2(A\,|L;H\,) \neq \sigma_2(A\,|L;L\,)$

The following theorem confirms that truthfully revealed messages followed by actions that depend meaningfully on the messages are no longer equilibrium profiles when only one player (player 1) talks.

Theorem 3 *With one-sided cheap talk where only player* 1 *talks, there does not exist an equilibrium with a responsive strategy profile where player* 1 *reports his type truthfully in the cheap talk stage, i.e.,* $a_1(H\,|H\,) = 1$ *and* $a_1(H\,|L\,) = 0$.

The proof of Theorem 3 has been postponed to the Appendix.

4.2 Cheap Talk with More Messages

Finally, we consider the implications of the players using a richer message space. So far, the only messages that the players were allowed to use at the announcement stage were their own types, i.e. $\mathcal{T}_i = \{L, H\}$. If instead, we allow the players to use more messages at the cheap talk stage, will that lead to new distinct equilibria that either have higher expected payoffs compared to the previous equilibria and/or that exist for values of p where the previous equilibria do not exist? We explore this interesting question by modifying our model and expanding the message space of each player.

Let each player i now choose an announcement τ_i' from the set $\mathcal{T}_i' = \{L, H\} \times \{X, Y\}$. We modify the strategies accordingly. An announcement strategy in the first stage for player i is a function $a_i : T_i \to \Delta(\mathcal{T}_i')$, where $\Delta(\mathcal{T}_i')$ is the set of probability distributions over $\mathcal{T}_i'$. So, for example, $a_i(HX \,|t_i)$ stands for the probability that strategy $a_i(t_i)$ of player i with type t_i assigns to the announcement HX. Beliefs for player i are now based on the new expanded message spaces. In the second (action) stage, a strategy for player i is a function $\sigma_i : T_i \times \mathcal{T}_1' \times \mathcal{T}_2' \to \Delta(S_i)$, where $\Delta(S_i)$ is the set of probability distributions over S_i.

We again restrict our attention to symmetric strategy profiles. The definition of a symmetric cheap talk equilibrium is similar to Definition 2 with the message spaces appropriately adjusted.

We consider a specific class of strategies where we impose the property that the cheap talk announcement should be fully revealing about their types. We modify Definition 3 in the following manner.

Definition 6 *A symmetric strategy profile* $((a_1, \sigma_1), (a_2, \sigma_2))$ *is called fully revealing if the announcement strategy* a_i *reveals the true types with certainty, i.e.,* $a_i(HX \,|H) + a_i(HY \,|H) = 1$ *and* $a_i(HX \,|L) = a_i(HY \,|L) = 0$.

We consider a symmetric fully revealing announcement strategy profile in which the H-type of player i announces HX with probability r_1 and HY with probability $(1 - r_1)$ and the L-type of player i announces LX with probability r_2 and LY with probability $(1 - r_2)$, i.e., $a_i(HX \,|H) = r_1$ and $a_i(LX \,|L) = r_2$. After the cheap talk phase, the possible message profiles (τ_1', τ_2') that the H-type of player 1 may receive are (HX, HX), (HX, HY), (HY, HX), (HY, HY), (HX, LX), (HX, LY), (HY, LX), (HY, LY), while the L-type of player 1 may receive (LX, HX), (LX, HY), (LY, HX), (LY, HY), (LX, LX), (LX, LY), (LY, LX), (LY, LY).

We now look at equilibria using such message profiles. The aim in studying such equilibria clearly should be to improve upon $S_{separating}$, the unique fully revealing symmetric equilibrium that was obtained with fewer messages (described in subsection 3.1), in which the players play the mixed strategy equilibria in the action stage when their types coincide, resulting in miscoordination and low payoffs. Using more messages may now help the players improve their coordination even when their types are the same.

In the new candidate equilibrium profile with more messages, we thus would like to keep the desirable type-coordination property when the types are different; for the type-coordination property to hold, we therefore restrict our attention to action-strategies where $\sigma_1(A \,|H; HX, LX) = \sigma_1(A \,|H; HX, LY) = \sigma_1(A \,|H; HY, LX) = \sigma_1(A \,|H; HY, LY) = 0$.This implies, again by symmetry, that $\sigma_1(A \,|L; LX, HX) = \sigma_1(A \,|L; LX, HY) = \sigma_1(A \,|L; LY, HX) = \sigma_1(A \,|L; LY, HY) = 1$.

However, note that in such an equilibrium if the messages are identical, then we are bound to have the mixed strategy equilibria in the action stage; to be an equilibrium, in the second (action) stage, by symmetry, an action-strategy of player 1 must have: $\sigma_1(A\,|H;HX,HX) = \sigma_1(A\,|H;HY,HY) = \frac{1}{1+H}$, $\sigma_1(A\,|L;LX,LX) = \sigma_1(A\,|L;LY,LY) = \frac{1}{1+L}$.

It is worth noting that the above partial specification of part of the players' strategies mimics $S_{separating}$. Let the class of strategy profiles satisfying the above partial specification be denoted by $S'_{separating}$.

The following result shows that new and distinct equilibria emerge by virtue of the richer message spaces used by the players.

Theorem 4 *Within the class of strategy profiles described by $S'_{separating}$, the following four profiles constitute fully revealing symmetric cheap talk equilibria:*

(i) $\sigma_1(A\,|H;HX,HY) = \sigma_1(A\,|L;LX,LY) = 0$, $r_1 = \frac{H^2}{1+H^2}$, $r_2 = \frac{L^2}{1+L^2}$;

(ii) $\sigma_1(A\,|H;HX,HY) = \sigma_1(A\,|L;LX,LY) = 1$, $r_1 = \frac{1}{1+H^2}$, $r_2 = \frac{1}{1+L^2}$;

(iii) $\sigma_1(A\,|H;HX,HY) = 0$, $\sigma_1(A\,|L;LX,LY) = 1$, $r_1 = \frac{H^2}{1+H^2}$, $r_2 = \frac{1}{1+L^2}$;

(iv) $\sigma_1(A\,|H;HX,HY) = 1$, $\sigma_1(A\,|L;LX,LY) = 0$, $r_1 = \frac{1}{1+H^2}$, $r_2 = \frac{L^2}{1+L^2}$.

All of the above equilibria exist when

$$\frac{L^4\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4} \leq p \leq \frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}.$$

Note that in each of the four profiles in Theorem 4, players are able to coordinate in the action stage on one of the pure equilibrium outcomes of the BoS, even when their types are the same by using different messages (X or Y).

In the Appendix, we provide the proof of (i). The other three cases are very similar and the proofs are therefore omitted.

We now compare the upper and lower bounds for p for the above equilibrium ($S'_{separating}$) and that of $S_{separating}$.

Note that

$$\frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3} - \frac{L^2+L^2H}{1+L+L^2+L^2H}$$

$$= \frac{L^2(H+1)(L+1)\left(H^3L+HL-L^2-1\right)}{(1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3)(1+L+L^2+L^2H)} < 0,$$

because $H^3L + HL - L^2 - 1 < L + L - L^2 - 1 = -(1-L)^2 < 0$.

This shows that the two intervals of p for the two equilibria ($S_{separating}$ and $S'_{separating}$) to exist are non-overlapping and the range for p for $S'_{separating}$ is strictly lower than the range for p for $S_{separating}$. Theorem 4 establishes that enriching the message space enables fully revealing equilibria to exist for values of p that were not possible otherwise.

5 CONCLUSION

In this paper, we have analysed a simple 2 x 2 x 2 Bayesian game and studied the possibility of information revelation and desirable coordination using one round of direct cheap talk. The main takeaway of our paper is that the desirable type-coordination is achieved at the unique fully revealing equilibrium (when it exists) moreover, such a coordination may also be achievable with partial revelation when fully revealing cheap talk equilibrium does not exist.

We here have characterised the unique fully revealing symmetric cheap talk equilibrium in the BoS with private information. There are of course many fully revealing but asymmetric cheap talk equilibria of this game. Clearly, babbling equilibria exist in which the players ignore the communication and just play one of the Nash equilibria of the complete information BoS for all type-profiles. There are other asymmetric equilibria as well, as Ganguly and Ray (2009) have already shown.

We are aware of many interesting open questions that come out of our analysis. For example, one may ask whether non-babbling or responsive cheap talk equilibrium always exist in our game for any given p or not. We also do not characterise the general case where neither type reveals truthfully in the cheap talk phase. Finally, following Banks and Calvert (1992), one may also be interested in characterising the ex ante efficient cheap talk equilibrium in our set up. We postpone all these issues for future research.

6 APPENDIX

We collect the proofs of our results in this section.

Proof of Theorem 1. As the strategies are symmetric, it is sufficient to characterise these candidate profiles only by $\sigma_1[A|H,L]$. There are only three possible candidates for $\sigma_1[A|H,L]$ as the complete information BoS with values H and L has three (two pure and one mixed) Nash equilibria. These profiles are (i) $\sigma_1[A|H,L] = \sigma^1(HL)$ where $\sigma^1(HL)$ is the probability of playing A in the mixed Nash equilibrium strategy of player 1 of the complete information BoS with values $t_1 = H$ and $t_2 = L$, that we call S_m; (ii) $\sigma_1[A|H,L] = 1$, that we call S_{ineff} and (iii) $\sigma_1[A|H,L] = 0$, which indeed is $S_{separating}$.

We first show that S_m is not an equilibrium. Under S_m, H-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)(\frac{H}{1+H}) \geq p(\frac{H}{1+L}) + (1-p)(\frac{H}{1+L})$, where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the corresponding optimal action strategy. This inequality implies $\frac{1}{1+H} \geq \frac{1}{1+L}$ which can never be satisfied as $H > L$.

The second candidate strategy profile, S_{ineff} is an equilibrium only when $\frac{1+H}{1+L+HL^2+L^2} \leq p$ and $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. To see this, note that under S_{ineff}, H-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p) \geq pH + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Similarly, L-type will announce his type truthfully only if $pL+(1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H})+(1-p)$ which implies $\frac{1+H}{1+L+HL^2+L^2} \leq p$. However, it can be shown that $\frac{1+H}{1+L+HL^2+L^2} > \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Hence, S_{ineff} cannot be an equilibrium.

Finally, we prove that $S_{separating}$ is an equilibrium only when $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Under $S_{separating}$, H-type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)H \geq p + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Similarly, L-type will announce his type truthfully only if $p + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)L$ which implies $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p$. ■

Proof of Theorem 2. For the strategy profile given in Theorem 2 (denoted by $S_{pooling}$) to be an equilibrium, we first need to check that $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$ are indeed consistent with the equilibrium conditions mentioned in the paper (in Section 3.2).

We first note that in the cheap talk phase, player 1(H-type) needs to be indifferent between announcing H and L. With the given values of $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$, the expected payoff from announcing H is $H(1-p)+p\left(H(1-r) + H\frac{r}{H+1}\right)$ while the expected payoff from announcing L is $pr + H(1-p)$ which will be equal when $r = \frac{H+H^2}{1+H+H^2}$.

Now, we observe the following:

if player 1(H-type) receives the message profile (H, L), then the expected payoff from playing A $(= 0)$ is less than the expected payoff from playing B $(= H)$, implying $q_1 = 0$;

if player 1(H-type) receives the message profile (L, H), then the expected payoff from playing A $(= 1)$ is greater than the expected payoff from playing B $(= 0)$, implying $q_2 = 1$;

if player 1(H-type) receives the message profile (L, L), then the expected payoff from playing A $(= \frac{p-rp}{1-rp})$ is less than the expected payoff from playing B $((1 - \frac{p-rp}{1-rp})H)$, implying $q_3 = 0$, only when $p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$, using $r = \frac{H+H^2}{1+H+H^2}$;

if player 1(L-type) receives the message profile (L, H), then the expected payoff from playing A $(= 1)$ is greater than the expected payoff from playing B $(= 0)$, implying $q_4 = 1$;

if player 1(L-type) receives the message profile (L, L), then the expected payoff from playing A $(= \frac{p-rp}{1-rp})$ is greater than the expected payoff from playing B $((1 - \frac{p-rp}{1-rp})L)$, implying $q_5 = 1$, only when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p$, using $r = \frac{H+H^2}{1+H+H^2}$.

Finally, in the cheap talk phase, it should be incentive compatible for player 1(L-type) to announce L. Using the inequality constraint (7), this requires $p \geq \underset{x}{Max}\,(1-p)\,((1-x)\,L) + p(r\frac{H}{1+H} + (1-r)\,((1-x)\,L))$, where x is the optimal probability of playing A in the action phase if player 1(L-type) deviates and announces H and receives the message profile (H, L). Again, note that the RHS of this last inequality constraint allows player 1(L-type) to deviate in both stages of the game. The derivative of the RHS of this inequality with respect to x is $L\,(p-1)+Lp(\frac{H+H^2}{1+H+H^2}-1) < 0$, which implies $x = 0$ and in turn shows that this condition is satisfied (LHS $= p \geq$ RHS $= \frac{\left(L+LH+LH^2+H^2p-LHp-LH^2p\right)}{1+H+H^2}$) only when $p \geq \frac{L+LH+LH^2}{1+H+LH+LH^2}$. Since $\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{L+LH+LH^2}{1+L+LH+LH^2} > 0$, the above gives us a meaningful range for p.

Hence, the profile constitutes an equilibrium if $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$. ■

Proof of Proposition 3. First, consider the case $p < \frac{L}{1+L}$. In this case, when the players' type profiles are (H, H), (H, L) or (L, H), the payoff to each player from $S_{separating}$ is obviously greater than that from the BNE because there is more miscoordination in the BNE in each of these states. When the type profile is (L, L), the payoff to each player from $S_{separating}$ is $\frac{L}{1+L}$ whereas the payoff from the BNE is $\left(\frac{1}{(1-p)(1+L)}\right)\left(1 - \frac{1}{(1-p)(1+L)}\right)(1 + L)$. Since $\frac{L}{1+L} - \left(\frac{1}{(1-p)(1+L)}\right)\left(1 - \frac{1}{(1-p)(1+L)}\right)(1 + L) = \frac{p(Lp-L+1)}{(1+L)(1-p)^2} > 0$, this proves that the expected utility from $S_{separating}$ ($EU_{separating}$) is greater than the expected utility from the BNE (EU_{BNE}).

Now, let $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$. Here, the structure of the BNE is such that the players fully miscoordinate when their true type profile is (H, H) or (L, L). This contrasts with $S_{separating}$ where the players achieve some degree of coordination in both the states (H, H) and (L, L). Since both $S_{separating}$ and the BNE achieve type-coordination, clearly $EU_{separating}$ is greater than EU_{BNE}.

Let us now consider $S_{pooling}$ and the BNE. In $S_{pooling}$, the players manage to coordinate with positive probability when the state is (H, H) but there is complete miscoordination when the true type profile is (L, L). Also, although there is type-coordination when the type profiles are (H, L) and

(L,H), the probabilities of these are different from that of the BNE because of partial revelation at the cheap talk stage.

When $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$, $EU_{BNE} = p\,(1-p)\,(1+H)$. Since $EU_{pooling} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$ and $\frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2} - p\,(1-p)\,(1+H) = \frac{p^2H(1+H)}{1+H+H^2} > 0$, $EU_{pooling}$ is greater than EU_{BNE}.

Finally, assume that $p > \frac{H}{1+H}$. Then,

$$EU_{BNE} = p^2\left(\left(\frac{H}{p(1+H)}\right)\left(1-\frac{H}{p(1+H)}\right)(1+H)\right) + p\,(1-p)\left(\frac{H}{p(1+H)}\right)(1+H)$$
$$= \frac{H}{1+H}.$$

So, in this case,

$$EU_{pooling} - EU_{BNE} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2} - \frac{H}{1+H}$$
$$= \frac{\left(p+3Hp+4H^2p+3H^3p+H^4p-H^3-H^2-H-p^2-2Hp^2-2H^2p^2-2H^3p^2-H^4p^2\right)}{(1+H+H^2)(1+H)}.$$

At $p = \frac{H}{1+H}$, it is easy to check that $EU_{pooling} - EU_{BNE} = H^3 > 0$.

Note that,

$$\frac{\partial}{\partial p}\left(EU_{pooling} - EU_{BNE}\right) = \frac{(1+H)\left(1+H+H^2-2p-2H^2p\right)}{1+H+H^2}$$

and

$$\frac{\partial^2}{\partial p^2}\left(EU_{pooling} - EU_{BNE}\right) = \frac{(1+H)\left(-2-2H^2\right)}{1+H+H^2} < 0.$$

So, $(EU_{pooling} - EU_{BNE})$ is strictly concave in p with a maximum value at $p = \frac{1+H+H^2}{2+2H^2}$,which is greater than the upper bound of the range of p for which $S_{pooling}$ exists (because $\frac{1+H+H^2}{2+2H^2} - \frac{H+H^2+H^3}{1+H+H^2+H^3} = \frac{1}{2}\frac{1-H^3}{1+H+H^2+H^3} > 0$). This shows that $(EU_{pooling} - EU_{BNE})$ is strictly increasing in the interval $(\frac{H}{1+H}, \frac{H+H^2+H^3}{1+H+H^2+H^3})$. Given that $EU_{pooling} - EU_{BNE} > 0$ at $p = \frac{H}{1+H}$, this proves that $EU_{pooling} > EU_{BNE}$ in the interval $(\frac{H}{1+H}, \frac{H+H^2+H^3}{1+H+H^2+H^3})$.

Hence, both $EU_{separating}$ and $EU_{pooling}$ are strictly greater than EU_{BNE}, whenever these simultaneously exist, confirming that cheap talk is strictly beneficial to both players. ■

Proof of Theorem 3. Let us first denote the action stage strategies by $\sigma_1(A\,|H;H) = s_1$, $\sigma_1(A\,|L;L) = s_2$, $\sigma_2(A\,|H;H) = s_3$, $\sigma_2(A\,|H;L) = s_4$, $\sigma_2(A\,|L;H) = s_5$, $\sigma_2(A\,|L;L) = s_6$. We divide all potential action strategy profiles into the following subcategories and prove that none of these can be an equilibrium.

Case (i): Both s_1 and s_2 are pure strategies

If $s_1 = s_2$, then this would imply that $s_3 = s_4 = s_5 = s_6$. This would then be a babbling strategy profile.

If $s_1 \neq s_2$, then it must be that $s_1 = s_3 = s_5$ and $s_2 = s_4 = s_6$. It is easy to check that in the first stage, one of the types for player 1 will deviate. For example, if $s_1 = 1$ and $s_2 = 0$, then player 1 (L-type) will want to announce that he is a H-type.

Case (ii): Both s_1 and s_2 are completely mixed strategies

For player 1(H-type) to be indifferent between playing A and B (where the LHS of the equation is the expected payoff from A and the RHS of the equation is the expected payoff from B), we must have:

$$ps_3 + (1-p)\,s_5 = p\,(1-s_3)\,H + (1-p)\,(1-s_5)\,H \tag{8}$$

For player 1(L-type) to be indifferent between playing A and B, we must have:

$$ps_4 + (1-p)\,s_6 = p\,(1-s_4)\,L + (1-p)\,(1-s_6)\,L \tag{9}$$

Also, in the cheap talk phase, it should be incentive compatible for player 1(H-type) to announce H, which implies

$$\begin{aligned} & p\,[s_1s_3 + (1-s_1)\,(1-s_3)\,H] + (1-p)\,[s_1s_5 + (1-s_1)\,(1-s_5)\,H] \\ \geq\ & \underset{s}{Max}\ p\,[ss_4 + (1-s)\,(1-s_4)\,H] + (1-p)\,[ss_6 + (1-s)\,(1-s_6)\,H] \end{aligned} \tag{10}$$

where s is the optimal probability of playing A in the action phase if player 1(H-type) deviates and announces L.

Using (8), the LHS of (10) $= p\,(1-s_3)\,H + (1-p)\,(1-s_5)\,H$. Note that the derivative of the RHS of (10) with respect to s is $ps_4 + (1-p)\,s_6 - p\,(1-s_4)\,H - (1-p)\,(1-s_6)\,H < 0$, which implies that $s = 0$. So, the RHS of (10) $= p\,(1-s_4)\,H + (1-p)\,(1-s_6)\,H$.

(10) $\Longrightarrow p\,(1-s_3) + (1-p)\,(1-s_5) \geq p\,(1-s_4) + (1-p)\,(1-s_6)$ which implies that $p\,(1-s_3)\,H + (1-p)\,(1-s_5)\,H > p\,(1-s_4)\,L + (1-p)\,(1-s_6)\,L$ and $ps_3 + (1-p)\,s_5 \leq ps_4 + (1-p)\,s_6$.

This contradicts (8) and (9).

Case (iii): <u>s_1 is a completely mixed strategy and s_2 is a pure strategy</u>

If $s_2 = 1$, then $s_4 = s_6 = 1$.

Then(10) $\Longrightarrow ps_3 + (1-p)\,s_5 \geq 1$ which can be satisfied only if $s_3 = s_5 = 1$ but that would mean that $s_1 = 1$ which is a contradiction.

If $s_2 = 0$, then $s_4 = s_6 = 0$.

Then(10) $\Longrightarrow p\,(1-s_3)\,H + (1-p)\,(1-s_5)\,H \geq H$ which can be satisfied only if $s_3 = s_5 = 0$ but that would mean that $s_1 = 0$ which is a contradiction.

Case (iv): <u>s_1 is a pure strategy and s_2 is a completely mixed strategy</u>

In the cheap talk phase, it should be incentive compatible for player 1(L-type) to announce L, which implies

$$\begin{aligned} & p\,[s_2s_4 + (1-s_2)\,(1-s_4)\,L] + (1-p)\,[s_2s_6 + (1-s_2)\,(1-s_6)\,L] \\ \geq\ & \underset{s}{Max}\ p\,[ss_3 + (1-s)\,(1-s_3)\,L] + (1-p)\,[ss_5 + (1-s)\,(1-s_5)\,L] \end{aligned} \tag{11}$$

If $s_1 = 1$, then $s_3 = s_5 = 1$.

Using (9), the LHS of (11) $= p(1-s_4)L + (1-p)(1-s_6)L$. Then(11) $\Longrightarrow ps_4 + (1-p)s_6 \geq 1$ which can be satisfied only if $s_4 = s_6 = 1$ but that would mean that $s_2 = 1$ which is a contradiction.

If $s_1 = 0$, then $s_3 = s_5 = 0$.

Then(11) $\Longrightarrow p(1-s_4)L + (1-p)(1-s_6)L \geq L$ which can be satisfied only if $s_4 = s_6 = 0$ but that would mean that $s_2 = 0$ which is a contradiction. ■

Proof of Theorem 4. For (i) to be an equilibrium, in the cheap talk phase, player 1(H-type) needs to be indifferent between announcing HX and HY. The expected payoff from announcing HX is $p\left(r_1\frac{H}{1+H} + (1-r_1)H\right) + (1-p)H$ while the expected payoff from announcing HY is $p\left(r_1 + (1-r_1)\frac{H}{1+H}\right) + (1-p)H$ which will be equal when $r_1 = \frac{H^2}{1+H^2}$.

Similarly, player 1(L-type) needs to be indifferent between announcing LX and LY. The expected payoff from announcing LX is $p + (1-p)\left(r_2\frac{L}{1+L} + (1-r_2)L\right)$ while the expected payoff from announcing LY is $p + (1-p)\left(r_2 + (1-r_2)\frac{L}{1+L}\right)$ which will be equal when $r_2 = \frac{L^2}{1+L^2}$.

For player 1(H-type), the expected payoff from announcing HX must be greater than or equal to the expected payoff from announcing LX, which implies

$$\begin{aligned} & p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)H\right) + (1-p)H \\ \geq \quad & p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)H\right) \end{aligned} \tag{12}$$

This will be satisfied if $p \leq \frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}$.

For player 1(H-type), the expected payoff from announcing HX must also be greater than or equal to the expected payoff from announcing LY, which implies

$$\begin{aligned} & p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right) + \left(1 - \frac{H^2}{H^2+1}\right)H\right) + (1-p)H \\ \geq \quad & p + (1-p)\left(\left(\frac{L^2}{L^2+1}\right)(1) + \left(1 - \frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right)\right) \end{aligned} \tag{13}$$

Note that $\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right) + \left(1 - \frac{L^2}{L^2+1}\right)H\right) - \left(\left(\frac{L^2}{L^2+1}\right)(1) + \left(1 - \frac{L^2}{L^2+1}\right)\left(\frac{H}{1+L}\right)\right) = L\frac{H-L}{L^2+1} > 0$. So, if the constraint (12) is satisfied, then this constraint (13) will also be satisfied.

For player 1(L-type), the expected payoff from announcing LX must be greater than or equal to the expected payoff from announcing HY, which implies

$$\begin{aligned} & p+(1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{L}{1+L}\right)+\left(1-\frac{L^2}{L^2+1}\right)L\right) \\ \geq \;& p\left(\left(\frac{H^2}{H^2+1}\right)(1)+\left(1-\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)\right)+(1-p)\,L \qquad (14) \end{aligned}$$

This will be satisfied if $p \geq \frac{L^4\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4}$.

For player 1(L-type), the expected payoff from announcing LX must be greater than or equal to the expected payoff from announcing HX, which implies

$$\begin{aligned} & p+(1-p)\left(\left(\frac{L^2}{L^2+1}\right)\left(\frac{L}{1+L}\right)+\left(1-\frac{L^2}{L^2+1}\right)L\right) \\ \geq \;& p\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)+\left(1-\frac{H^2}{H^2+1}\right)L\right)+(1-p)\,L \qquad (15) \end{aligned}$$

Note that $\left(\left(\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)+\left(1-\frac{H^2}{H^2+1}\right)L\right)-\left(\left(\frac{H^2}{H^2+1}\right)(1)+\left(1-\frac{H^2}{H^2+1}\right)\left(\frac{H}{1+H}\right)\right)=-\frac{H-L}{H^2+1}<0$. So, if the constraint (14) is satisfied, then this constraint (15) will also be satisfied.

This implies that the strategy profile given by (i) is an equilibrium if $\frac{L^4\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4} \leq p \leq \frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}$.

Finally, note that
$\frac{HL^3\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3}-\frac{L^4\left(1+H+H^2+H^3\right)}{1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4}$
$=\frac{L^3(H-L)\left(1+H+H^2+H^3\right)\left(1+L+L^2+L^3\right)}{(1+L+L^2+L^3+HL^3+H^2L^3+H^3L^3+H^4L^3)(1+L+L^2+L^3+L^4+HL^4+H^2L^4+H^3L^4)}>0$

So, this gives us a legitimate interval for p. ■

7 REFERENCES

1. Aumann, R. and S. Hart (2003), "Long cheap talk," *Econometrica*, 71, 1619-1660.

2. Baliga, S. and S. Morris (2002), "Coordination, spillovers and cheap-talk," *Journal of Economic Theory*, 105, 450-468.

3. Baliga, S. and T. Sjöström (2004), "Arms races and negotiations," *Review of Economic Studies*, 71, 351-369.

4. Baliga, S. and T. Sjöström (2012), "The strategy of manipulating conflict," *American Economic Review*, 102, 2897-2922.

5. Banks, J. S. and R. L. Calvert (1992), "A battle-of-the-sexes game with incomplete information," *Games and Economic Behavior*, 4, 347-372.

6. Burton, A., G. Loomes and M. Sefton (2005), "Communication and efficiency in coordination game experiments," in J. Morgan (ed.), *Experimental and Behavioral Economics*, Volume 13 of *Advances in Applied Macroeconomics*, JAI Press, 63-85.

7. Cabrales, A., W. García-Fontes and M. Motta (2000), "Risk dominance selects the leader: an experimental analysis," *International Journal of Industrial Organization*, 18, 137-162.

8. Camerer, C. F. (2003), *Behavioral game theory*, Princeton University Press.

9. Chen, Y (2009), "Communication with two-sided asymmetric information," *Mimeo.*

10. Cooper, R., D. V. DeJong, R. Forsythe and T. W. Ross (1989), "Communication in the battle of the sexes game: some experimental results," *Rand Journal of Economics*, 20, 568-587.

11. Costa-Gomes M. (2002), "A suggested interpretation of some experimental results on preplay communication," *Journal of Economic Theory*, 104, 104-136.

12. Crawford, V. P. (1998), "A survey of experiments on communication via cheap talk," *Journal of Economic Theory*, 78, 286-298.

13. Crawford, V. P. and J. Sobel (1982), "Strategic information transmission," *Econometrica*, 50, 1431-1451.

14. Dixit, A. and C. Shapiro (1985), "Entry dynamics with mixed strategies," in Thomas, L. G. (ed.), *The Economics of Strategic Planning*, Lexington Books.

15. Doraszelski, U., D. Gerardi and F. Squintani (2003), "Communication and voting with double-sided information," *Contributions to Theoretical Economics*, 3, Article 6.

16. Farrell, J. (1987), “Cheap talk, coordination, and entry,” *Rand Journal of Economics*, 18, 34-39.

17. Farrell, J. and R. Gibbons (1989), “Cheap talk can matter in bargaining,” *Journal of Economic Theory*, 48, 221-237.

18. Farrell, J. and G. Saloner (1988), “Coordination through committees and markets,” *Rand Journal of Economics*, 19, 235-252.

19. Fudenberg, D. and J. Tirole (1991), *Game theory*, MIT Press.

20. Ganguly, C. and I. Ray (2009), “Unmediated and mediated communication equilibria of battle of the sexes with incomplete information.” In Basu, B, B. K. Chakrabarti, S. R. Chakravarty and K. Gangopadhyay (eds.), *Econophysics & Economics of Games, Social Choices and Quantitative Techniques*, Springer, 347-361.

21. Ganguly, C. and I. Ray (2013), “Information-revelation and coordination using cheap talk in a Battle of the Sexes with two-sided private information.” 13-15 (13-01R), Discussion Papers, Department of Economics, University of Birmingham.

22. Goltsman, M. and G. Pavlov (2014), “Communication in Cournot oligopoly,” *Journal of Economic Theory*, 153, 152-176.

23. Horner, J., M. Morelli and F. Squintani (2015), “Mediation and peace,” *Review of Economic Studies*, 82, 1483-1501.

24. Krishna, V. and J. Morgan (2004), “The art of conversation: eliciting information from experts through multi-stage communication,” *Journal of Economic Theory*, 117, 147-179.

25. Matthews, S. A. and A. Postlewaite (1989), “Pre-play communication in two-person sealed-bid double auctions,” *Journal of Economic Theory*, 48, 238-263.

26. Moreno de Barreda, I. (2012), “Cheap talk with two-sided private information,” *Mimeo.*

27. Park, I.-U. (2002), “Cheap talk coordination of entry by privately informed firms,” *Rand Journal of Economics*, 33, 377-393.

28. Seidmann, D. (1990), “Effective cheap talk with conflicting interests,” *Journal of Economic Theory*, 50, 445-458.