Media bias in the best and worst of times

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Abstract

Readers of news have preference for news sources which are closer to their beliefs. I use this fact to develop a Hotelling's linear city model of competition between two ideologically different media firms which are supplying information about a given topic. While reporting any topic, media fulfils its two motives- ideology payoff and better assessment of its news from readers. Readers assess news based on their own ideology and the facts related to the topic. I find that competition can lead both firms to provide accurate information regarding some topics but deviate from each other while reporting others. If the topic is unfavorable to a media's ideology, then it either reports to defend its ideology, implying a detachment of its ideology from the topic or delivers a closely accurate report of the unfavorable event. Interestingly, when the topic is unfavorable, the media refrains from reporting in an indifferent manner. By the standard Hotelling's result, readers incur a transportation cost when they read news distant from their own ideology. In the current model, I show conditions when such results fail to hold such that readers give better assessment to news of a media placed farther away from their ideology than one which is nearer. In the absence of competition, welfare decreases as the media gains license to bias news regarding unfavorable topics. On the other hand, the entry of a third firm does not necessarily enhance the welfare levels of the economy. Policy prescriptions like educating readers to stress more attention on facts can lead to readers willingly accepting an accurate report about a topic which carries opposite ideology.

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1 Introduction

Mainstream media is a vital institution in any democracy which gathers and disseminates information from all spheres of social and political life to the public. In this process, they exercise great power in establishing public opinion, creating consensus, affecting electoral outcomes and increasing political involvement. Partisanship is a major factor which directs media to take these various roles and have attracted scholarship across economics, political theory and communication theory.

Partisanship of media mainly surfaces as bias in news. News bias refers to the tendency of media firms to deviate from the truth while reporting any topic. For instance, the onset of higher inflation under a leftist government can lead a left media to defend the government's policies by highlighting on the policy's unemployment improving outcomes. Now why might a media bias news? (Hamilton, 2011) posits media news not as mere information, but as a commercial product, shaped up by both supply-side forces (ownership, ideological affiliations, nature of topic) and demand-sided factors (reader beliefs).

The importance of understanding media bias lies in the power of biased information to change electoral outcomes, reinforce public opinions or generate policy changes within an economy [1]. This effect of biased information gains further momentum as a considerable section of the population still rely on media firms to learn about world events. The survey of Smith and Lichter (1997) shows 82% of the participants believed that media must be the foremost news provider. In addition, 75% strongly assert media to take the role of watchdogs on public officials to curb their intentions to abuse power[2].

Existing literature studying media bias under competition can be segmented under two broad groups - supply-driven and demand-driven. The supply-side analysis shows that news bias is lowered with increased competition which leads to greater welfare. On the other hand, the demand-sided models consider that consumers prefer news which confirms their prior beliefs. These models focus on various motives of competing media firms like ideology, reader attention, advertisement revenues, electoral motives.

However, the existing literature do not account for two primary features. The first relates to modelling competition for attention. Most models view attention as an economic unit which reaps profits. However, gaining greater reader attention in time dimension does not necessarily entail better acceptance of news to a reader. For instance, a conservative media might have the attention of a liberal reader, but it is not certain whether he would approve its merit above a liberal media. In this model, therefore, the reader chooses to attach a weight on the news story which

[1] See Chan and Suen (2008), Duggan and Martinelli (2011),Prat and Strömberg (2013)

[2] Evidences about reader's mistrust on news media is provided by Gentzkow, Shapiro and Stone (2015) where news consumers feel media to be biased or produce news which counters their ideology.

indicates how close he is to accept it. Secondly, my model analyses whether partisan media can reap ideology payoffs by biasing information about factual reports already available to readers. In the current model, readers do not depend on media to receive information about an uncertain state, but want to get informed from a media which aligns with his beliefs. Laura Silver (2021) shows the reluctance of the public to accept plain factual information, which lays the ground the current analysis. Also, acceptance of facts vary across ideological lines[3] which adds a non-trivial role of studying news bias due to competition between media with different ideologies. [4] This appeals to the literature on biased perception to factual information and in this regard, I provide a simple theoretical measure of reader assessment of news. This measure indicates when partisan-motivation overrides the motivation for accurate information and vice-versa. I examine how such variance in perception can act as a leverage or as an impediment to reach the partisan goals of a media under conditions like reader polarization, reader sophistication, motives of the rival media and the event at hand. I later propose policy recommendations based on this measure which show conditions where accurate information can be released which gains greater acceptance from more ideologically-oriented readers.

The present model begins with an honest information source which provides the reader populace with a factual report about a topic. Examples of such sources include Supreme court, Bureau of Labor Statistics (BLS), Associated Press, Reuters who are known to '.. represent the essence of objective news coverage, as they self-consciously avoid politically based editorial judgments in their news content", Baum and Groeling (2008). Following this, two ideologically opposite media firms (left and right) media compete on a spatial ideology spectrum to inform the reporters further about the topic. Consider for instance that the readers learn the last-month unemployment rate is at 5-percent. How can partisan media bias this report? Under leftist presidency, the left media can report how it has decreased from a higher rate while under a rightist presidency, it can report how it has increased from a previous rate. [5] The right media will also take analogous measures. The both ends of the ideology spectrum along which the media compete resemble the extreme left and right ideologies. Readers are heterogeneous and their location on this spectrum denote their closeness to either ideology. My results hinge on two basic assumptions in this model. First, readers are ideological and read news after learning a factual report which creates a point of reference while assessing news. Second, media cares about its own ideology and maximising reader assessment. Given this, the present model leads to some interesting insights. Readers read news from both

[3](Schaffner and Roche (2016),Jerit and Barabas (2012)) studies how factual reports related to inflation and unemployment are unevenly processed by Republican and Democratic readers. Bartels and Johnston (2013) coins the phrase 'subjective ideological disagreement to describe how Supreme Court's policymaking if liberal-oriented can lose its legitimacy to the more conservative populace and vice versa.

[4]Ognianova and Endersby (1996) uses survey data prior to 1974 election showing that reader perception of politicians is linked with their partisanship and their evaluation of news media.

[5]This example is taken from Groseclose and Milyo (2005).

media but might not accept the news equally. This acceptance is governed by their own ideology and the prior knowledge of the event. For, instance a left reader might discredit news of media R simply based on ideology differences.

First, competition among rival media firms mitigate or exacerbate the level of bias depending on the relative weights given towards ideology and reader assessment. Second, for particular topics and certain parameter values, not biasing news leads to lower profits. Third, if media tries to defend its ideology by countering the factual report, then its profits will dwindle. When the factual report stands contrary to the media's ideology, then media will refrain from taking an indifferent stance - it will either defend its ideology, by disassociating its ideology from the event or it reports become consistent with the factual report. These patterns are dictated by the weight it assigns its two motives- ideology and reader-assessment. Fourth, a novel measure of reader-satisfaction is provided and under specific conditions, readers can gain relatively more utility from news which is farther away from their beliefs than one which lies closer. Fifth, media receives greater leeway to bias news in its favor when the audience is more unsophisticated, who are less educated and tolerates bias [6]. However, the impact on bias from higher reader unsophistication gathers force when both the media firms are more focused towards ideology gains than gains from reader-assessment. Sixth, welfare is not necessarily enhanced in presence of media firms which care more for reader-assessment. Welfare is dependent on the number of readers in the economy and how they are spread across the ideology spectrum.

Before proceeding with my model, I briefly layout the main forms of media-bias. Following Puglisi and Snyder Jr (2011), news bias by a partisan media mainly occurs in three forms - selective reporting (reporting on strongly partisan topics); issue framing (how an event is portrayed by reporters)[7] and 'agenda setting' (determined by amount of coverage on each incident). In the present setup, bias takes the form of issue-framing and is generated by both demand (reader-assessment) and supply (ideology of media firm) factors. Since competition between media is spatial, the placement of news is a single point on the ideology spectrum which represents the bottom line or a condensed form of the event. The location of this point then signifies how close a particular media has chosen to be to the left or right ideology.

In later sections, the results of the benchmark model of duopoly competition is compared with respect to three settings - monopoly media (absence of competition), more polarized reader distribution (for instance, when majority readers are biased to the left or to the right) and a market with three firms. I find equilibrium bias increases in the monopoly setup, due to absence of any competition. In the three-media case, if the event to be reported is has no ideology bearings, then

[6]Prat (2018) discusses a measure of media power which gains force as reader sophistication tends to zero.

[7] (Mullainathan and Shleifer, 2002) dissects these two as 'bias' and 'spin', the former being in context of traditional left-right ideology while the latter helps to create a memorable story

equilibrium media bias rises above the duopoly bias level only if the third firm is ideologically biased. I then analyse welfare by aggregating reader utility and media firms payoffs. I find welfare to depend crucially on the nature of topic to be reported, the weights assigned by a media to its twin motive - ideology and reader-assessment and reader polarization.

The remaining of the paper proceeds in the following fashion: section 2 the related theoretical and empirical literature. Section 3 introduces the model preliminaries; the game timeline has been laid out in section 4; section 5 examines the duopoly competition; section 6 analyses media bias in presence of a more polarized reader pool; section 7 provides a brief insight into the outcomes when a third media enters the duopoly market and section 8 presents the welfare analysis.

2 Related Literature

This paper fits in the literature of industrial organizational aspect of media bias. As discussed before, I model a competitive model of media bias as a product placement problem. While reporting certain issues, rival firms try to place themselves closer to each other whereas they maximally differentiate from each other while reporting on others. This model does not account for media's role either in electoral outcomes or policy analysis (Chan and Suen (2008), Bernhardt, Krasa and Polborn (2008), Duggan and Martinelli (2011)), Prat and Strömberg (2013)) or in models of media capture like Besley and Prat (2006).

Nevertheless, among these papers we find support of the model premises. The manner in which we define bias matches with Duggan and Martinelli (2011),D'Alessio and Allen (2000) or Mullainathan and Shleifer (2002). This definition directs us to a specific strand of works within the spatial product-placement literature of Anderson and McLaren (2012), Chan and Suen (2008) and Bernhardt, Krasa and Polborn (2008). These papers however work at the conjunction of media bias and its extensions in various political and electoral environments.

The structural aspects of Mullainathan and Shleifer (2002) closely resonates with this model where readers learn about the issue before reading media reports. However, both differ in other underlying model assumptions and the nature of news provision. A finding common to both is the information slant by media about reports on events with no ideology. They argue that the bias exist through the channel of 'spin' which creates a memorable story whereas my argument depends on a result following Hotelling's lemma, where each partisan media outlet segment the economy and bias the ideology-free event to cater to their like-minded readers. In addition, this model offers an added insight which can be explained through the following example. Consider two scenarios - A, where both media firms refuse to compromise with their partisan interests and B where both are relatively flexible about adjusting partisan priorities to satisfy readers. Then while reporting a neutral incident, media firms in A will earn relatively higher equilibrium profits than the ones in

B. Apart from this, I add to the literature, by providing a formalized way to detect when media firms will speak indifferently and when they will not.

This paper is also close to the literature focusing on media bias from non-price competition in a duopoly setup which includes that of (Gentzkow and Shapiro, 2006) who study a supply-side story of media bias, where reputational concerns of media drives it to take certain editorial choices. They show exogenous chance of truth revelation disciplines media and prevents it form biasing news. (Bernhardt, Krasa and Polborn, 2008) show that media outlets seeking to maximize profit take sides and introduce bias to their stories which later lead to voters committing electoral mistakes.

In contrast to the above models, the present model characterization implies how media will locate itself on a spatial axis when it has to inform the public about it. Media's choice depends on its valuation on reader evaluation and how favourable the incident it relative to its own ideology. I try to throw clarity when media will proclaim the superiority of its own ideology beyond the truth or trivialize or report partially a ideologically 'bad' event or sound indifferent. Additionally, I put forward the associated profit levels with these media choices and infer how media outlets are affected while choosing their news stories.

2.1 Empirical Implications

The theoretical findings are close to the results from number of empirical papers surrounding media bias. I also find support for the underlying model premises which I first lay out. Firstly, I assume media's profit as a function of partisan gains and gains from reader evaluations. The importance of partisan gain is supported by the supply-side estimation in Gentzkow and Shapiro (2010) where media's slant responds to customer ideology and their owner's type. Further empirical support behind partisan features of media are established in Budak, Goel and Rao (2016). The demand-side estimations from Gentzkow and Shapiro (2010) supports the importance of reader evaluations where they find that consumers try to match their own ideology with the media's slant which substantiates the logic behind the inclusion of this factor in the profit function. The latter assumption finds its ground in (Iyengar et al., 1984) whose experimental findings suggest that reader evaluations based on media news are indeed instrumental in the ambit of political consequences. The point where we deviate from the above works is the assumption that readers and media already know the reality through fact-based reporting. Apart from theoretical support (Mullainathan and Shleifer, 2002), empirical evidences are found in Iyengar et al. (1984) and Higgins, Rholes and Jones (1977). These papers state that the presence of such coverage not only provokes readers to recollect memories on a previous event, but also plants an initial comprehension of the same in the reader's mind, based on which he forms his evaluations regarding the current news. The prior information in essence leads the reader to judge information differently in context to his initial comprehension.

The structure of media bias as a product differentiation model is found in Hamilton (2011) who states that readers and media firms can be mapped on an ideology spectrum and readers deem a media as biased depending on how far it is from his ideology. However, the current paper adds the role of a factual report which makes the reader match a news story with his own ideology as well as with the factual report. The concept of reader-assessments of news is also found in Hamilton (2011) (p. 74) where it is stated that an economy comprised of mostly liberal public will deem the news of more liberal-oriented news as not biased. This is one of the results in section 6 which analyses the behavior of partisan media in a biased reader pool. The effect of biased reader pool on the degree of media slant is also found in Gentzkow and Shapiro (2010) who shows a statistically significant rise in slant in presence of like-minded public. However, my analysis also provides the way readers perceive such news which ultimately affects media profits. We show that strongly idealistic readers may deem a news story of their like-minded news channel as unsatisfactory which might be due to the absence of enough ideology slant.

The model also sheds light on the tendency of media to take indifferent stances on a range of issues has been supported by anecdotal evidences which has been cited in proposition 3. The theoretical observation that under certain conditions, a reader may prefer news from an ideologically-opposite media channel only if the incident supports their ideology is found in the experimental evidences of (Kuklinski and Hurley, 1994). If the incident seems harmful towards their beliefs, then readers prefer their like-minded channel.

3 Model

This fundamental model is akin to the linear city model where readers are uniformly distributed. In the baseline case, there are two partisan media firms L and R at opposite ends which signifies their ideological rivalry. The entire game spans across three periods. In the first period, readers receive an exogenous factual report by a honest media E about a particular event ω belonging to the universe $\Omega = [-1, 1]$. The point -1 represents an incident which aligns to the extreme left ideology while 1 denotes an event with extreme rightist ideology. Any intermediate points are relatively moderate and the midpoint 0 is absolutely neutral.

The above interval also represents the linear city where $N \in \mathbb{N}$ readers are uniformly [8] placed and their location also denotes their subjective ideological leanings. A reader i's position is denoted as x_i on $[-1, 1]$. The neutral (or moderate) reader is positioned at 0 while the extreme leftist (rightist) reader is placed at -1 (1) as shown in Figure 1. Readers are rational and are aware of the

[8] Cumulative mass function F and probability mass function f.

partisan interests of media. The utility of a reader i is additively separable [9] across news of media $j \in \{L, R\}$ is

$$U_{ij} = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2 \tag{1}$$

The action of i is to choose $\alpha_{ij} \in \mathbb{R}$ which denotes his assessment or weight of the news story by media j. Intuitively, this is a measure of the degree of satisfaction from a news story. This assessment is therefore a mapping $\alpha_{ij} : \theta_E \times \theta_j \rightarrow \mathbb{R}$, where θ_{-j} refers to the strategy or editorial position of the rival media and θ_E denotes the signal from the honest media E.

In the following period, media firms L and R choose their respective editorial positions θ_L and θ_R on the same ideology interval $[-1, 1]$ on the onset of a particular event θ_E. The baseline model accounts for media firms initially located at the extremes. This is akin to the concept of bliss point or where the partisan media firms ideally want to be. We parameterize this location by $\bar{\theta_j} \in [-1, 1]$. In the baseline model, $\bar{\theta_L} = -1$ and $\bar{\theta_R} = 1$. So my aim is to understand how information bias percolates into an economy when its news suppliers are inherently extreme partisans. One can do the same for other moderate values of $\bar{\theta_j}$ and examine the levels of information slant.

The payoff function of media j accounts for the action of its rival firm ($-j$), given the report of the honest media as shown below.

$$\Pi_j(\theta_j, \theta_{-j}|\theta_E) = -\lambda_j.(\alpha_j^* - 1)^2 - (1 - \lambda_j)(\theta_j - \bar{\theta_j})^2 - c\frac{(\theta_j - \theta_E)^2}{b + (\theta_{-j} - \theta_E)^2} \tag{2}$$

The action of media j is a mapping θ_j where $\theta_j : \theta_{-j} \times \theta_E \rightarrow \mathbb{R}$ where θ_{-j} denotes strategy of the rival outlet. The first two terms depicts the trade-off between accuracy and ideology to media j respectively. Hence, media's payoff is a convex combination of these two factors with respective weights λ_j and $(1 - \lambda_j)$ where $\lambda_j \in (0, 1)$. The first term implies gaining better reader assessment [10] while the second term denotes the gain in ideology payoff by locating closer to its ideology bliss point $\bar{\theta_j}$. If λ_j is very closer to 1 then j places greater weight on reader satisfaction. On the contrary, when λ_j is closer to 0, media j weighs ideological gains more than reader satisfaction.

The final term denotes the cost function $C(.)$ of j from biasing news which is basically the deviation of θ_j from θ_E. The marginal cost is $c > 1$. The parameter $b \in (0, 1)$ represents the sophistication within the readers. This has a cross-over effect of one firm's bias on its rival. Higher (lower) value of b implies lower (greater) cost of bias, given the level of bias of the other firm. $C(.)$ has the following properties:

[9] This convention has been used in Gentzkow and Shapiro (2010) with a more ordinal utility form, where a household's utility is additive in the number of newspapers chosen among the ones available within its zip code.

[10] We explain this functional form more clearly using Lemma 1 in section 5.

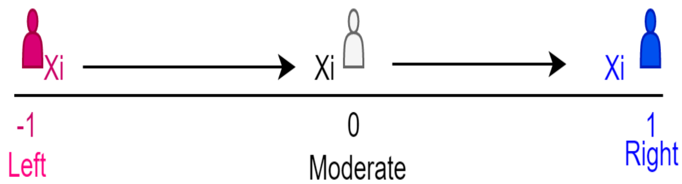

Figure 1: Location of readers

(i). $\frac{dC}{d(\theta_j - \theta_E)^2} > 0$, firm j incur greater cost by biasing news.

(ii). $\frac{dC}{d(\theta_{-j} - \theta_E)^2} < 0$, firm j faces lower cost from biasing when its rival firm biases news and vice versa.

(iii). $\frac{dC}{db} < 0$, cost of bias decreases when level of reader un-sophistication increases.

I solve this duopoly game Γ_D of complete information using Subgame Perfect Nash Equilibrium (henceforth SPNE).

Definition 1. *A strategy profile* $s = \{\theta_L, \theta_R, < (\alpha_{1L}, \alpha_{1R}), .., (\alpha_{NL}, \alpha_{NR}) >\}$ *of* Γ_D *is a subgame perfect Nash equilibrium (SPNE) if* s *induces a Nash equilibrium in every subgame of* Γ_E*. Nash Equilibrium of the duopoly game* (Γ_D) *between the media is a pair* (θ_L^*, θ_R^*) *of editorial choices for which* θ_L^* *is a best response to* θ_R^* *and* θ_R^* *is a best response to* θ_L^*.

4 Timeline of game

Figure 2 illustrates the timeline of this model which begins with a naturally occurring event ω, through some exogenous random process. An unbiased media E sends a factual public report θ_E which becomes common knowledge to both readers and partisan media. Mullainathan and Shleifer (2002) refers this as a signal 'r'which sets a prejudice within a reader before he reads the news. The partisan media firms L and R observes the event and θ_E and designs its own report (reflected by its editorial positions θ_L and θ_R respectively) for the readers in the next stage. Readers are heterogeneous and rational and they cannot observe the true event prima facie but has access to the news of E. After the partisan media publishes the report, readers assess its report and provides a rating (α_L to L; α_R to R) which measures the report's consistency with E's report and their subjective ideology. These ratings can act as instruments to measure the unrest or ecstasy among the readers about any particular event. I study the editorial decision of partisan media through a simple backward-induction game in a duopoly media market. Media firms L and R compete over attention along a spatial Hotelling's axis which measures ideology.

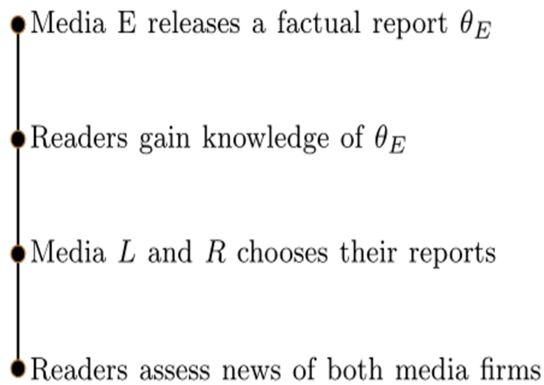

Figure 2: Timeline of the duopoly game

For further clarity, I explain the timeline using a simple example. After learning about a potential deportation through a graduate school email (following the online mode of classes in Fall 2020 due to Covid-19), international students will likely read reports of say, CNN and Fox to gather more information. Here, the graduate school resembles media E which presents a factual report. Students (readers) then can tune to CNN to hear its condemnation against the Immigration and Customs Enforcement (ICE) for imposing such a strategy, thereby gathering some solace after feeling victimized. Alongside, they might also tune to Fox to learn how likely they are to be deported. These experiences from a liberal and conservative media are portrayed by the reader-ratings (α_L and α_R respectively).

5 Duopoly Model

I consider the duopoly media market with firms L and R. Then the corresponding normal form game of this duopoly case is defined as

$$\Gamma_D = [I, \{u_i(.)\}, \{\Pi_L(.)\}, \{\Pi_R(.)\}]$$

I denotes the player set comprising of media L and R and reader $i \in \{1, .., n\}$. u_i is the utility of reader i from reading news and Π_L and Π_R denotes the profits of media L and R. Thereby the equilibrium strategy profile constituting the SPNE is characterized as $s^* = (\theta_L^*, \theta_R^*, \alpha_{iL}^*(\theta_L^*), \alpha_{iR}^*(\theta_R^*))$.

5.1 Utility maximization of reader

First order condition following equation 1 leads to the optimal assessment (weight) given by reader i towards media j's editorial position θ_j

$$\alpha_{ij}^* = \frac{x_i + \theta_E}{2\theta_j} \tag{3}$$

Lemma 1. *The first best evaluation by reader i reading news of media $j \in \{L, R\}$ is achieved when $\theta_j = \theta_E = x_i$, or*

$$\alpha_j^* = 1$$

The rating of 1 suggests that media j's editorial position matches both media E's position θ_E and the ideology x_i on $[-1, 1]$ in tandem. Intuitively, if $\theta_j = \theta_E = x_i$ then not only does i perceive j to be as honest and accurate as E, but also can relate it perfectly with his own ideology x_i. Hence this news is perfectly cohesive with his rational self.

The first term in the profit function of j is a distance function which accounts for the loss of reader satisfaction from a piece of news which cannot be assigned this first-best weight.

Borrowing equation 3, the expected rating from N readers of j is given as

$$E(\alpha_j^*) = \sum_{i=1}^{N} \frac{x_i + \theta_E}{2\theta_j}.f(x_i) = \sum_{i=1}^{N} \frac{x_i + \theta_E}{2\theta_j}.\frac{1}{N} = \frac{\theta_E}{2\theta_j} \tag{4}$$

The above result arrives from the assumption that readers are distributed such that mass of leftist and rightist readers are equal, hence they offset each other ($\sum_{i=1}^{i=N} x_i = 0 \quad \forall i \in \{1, .., N\}$). $\sum_{i=1}^{i=N} x_i \neq 0$ implies more polarised readers such that the distribution of readers f is such that the mass of leftist readers either greater or lesser than their rightist counterparts. If $\sum_{i=1}^{i=N} x_i \leq 0$, ($\sum_{i=1}^{i=N} x_i \geq 0$) the economy has a leftist (rightist) majority. The impact of such an unbalanced reader base on the editorial positions has been explored in section 6.

5.2 Payoff maximization of media

The optimal action of media j is directed by the below first-order-condition

$$\frac{d\Pi_j}{d\theta_j} = \theta_j^4 \left[(1 - \lambda_j) + \frac{c}{(b + (\theta_{-j} - \theta_E)^2)} \right] \quad - \theta_j^3 \left[(1 - \lambda_j)\bar{\theta}_j - \frac{c\theta_E}{b + (\theta_{-j} - \theta_E)^2} \right] \tag{5}$$
$$+ 0.5\lambda_j \theta_E \theta_j - 0.25\theta_E^2 \lambda_j = 0$$

This represents the best response function of j to the action of its rival θ_{-j}. The equilibrium editorial choice(s) is attained at the intersection of these functions. To bring out the possible

Figure 3: The blue (orange) segment denotes class of events which supports the mixed-strategy equilibrium of L (R) at a particular threshold of λ_L (λ_R). These thresholds are unique for every event $\theta_E \in [\bar{\theta}_E^R, \bar{\theta}_E^L]$. Events outside these area support unique equilibrium of for all values of λ_L or λ_R

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behavior traits of media, I limit the value of b to be above some threshold as stated in Assumption 1. It is only above a cutoff that the effects of media under this setup becomes pronounced enough for a deeper analysis.

Assumption 1. *b is above a threshold level $b' \in (0, 1)$.*

This threshold value can act as a direct measure of reader un-sophistication and finds support in the experimental findings of (Iyengar et al., 1984) who posits that experts are much less influenced by manipulations by media and have already established their own evaluations about a particular event. On the other hand novices are the vulnerable ones, totally non-immune to information manipulations by media. L faces much higher cost in the event when rival media R does not bias. As b increases, it allows L to bias news and insulates against any negative feedback from the public. This simultaneously weakens competition to publish more accurate information and exacerbates the level of information slant.

Before proceeding into the equilibrium properties, it must first be ensured that the above system of equations have at least one real root within the interval of interest i.e $[-1, 1]$. Given the quartic nature of equation 5, it is close to impossible to postulate an explicit solution for θ_j. However, using *Sturm's Theorem*, it is suggested that two real solutions exists in $[-1, 1]$, as proposed by *Lemma 2*. For any parameter values, each polynomial has two real roots within $(-1, 1)$, one positive and one negative. I provide detailed explanation about this rule in section 8.1 of the appendix.

Lemma 2. *There exists two distinct real roots (one positive, one negative) in $(-1, 1)$ of the best response function of each media.*

The following proposition describes the conditions which support both pure and mixed strategy equilibrium. In equilibrium, the BR functions intersects providing the associated profit levels to each media firm.

Proposition 1. *(i)* ***Pure strategy equilibrium:*** *For any λ_j, θ_j^* is unique for any $\theta_E \in \{[\bar{\theta}_E^R, 0) \cup (0, \bar{\theta}_E^L]\}^C$. However, for $\theta_E \in [\bar{\theta}_E^R, 0) \cup (0, \bar{\theta}_E^L]$, θ_j^* is unique for any $\lambda_j \neq \bar{\lambda}_j$.*

(ii) ***Mixed strategy equilibrium:*** *For a class of events lying in $[\bar{\theta}_E^R, 0)$ and $(0, \bar{\theta}_E^L]$, there exists a mixed strategy equilibrium of L and R at a unique cutoff value of $\bar{\lambda_L}$ and $\bar{\lambda_R}$ respectively. Here the equilibrium strategy pair for each media $j \in \{L, R\}$ is denoted by $(\theta_j^{1*}, \theta_j^{2*}; p, 1-p)$ and both lie on either side of zero.*

(iii) ***Symmetric equilibrium:*** *When $\theta_E = 0$, a symmetric equilibrium exists when $\lambda_L = \lambda_R$ when L and R positions themselves equidistant from the median reader at 0.*

The first two statements can be understood with more clarity through figure 3. Events to the right of the blue interval support the right ideology strongly enough such that L always locates on the right of 0 for all values of λ_L. This is unique pure strategy equilibrium for both L and R. Symmetric results evolve for event to the left of the blue interval. Compared to this, events in the blue (orange) intervals favor the left (right) relatively with lower magnitude. Then, reporting in favor of the left for events in the blue interval is no longer binding for R unless when λ_R is high enough (greater weight on reader assessment). The model provides a cutoff $\bar{\lambda_R}$ which determines the equilibrium response of R. below which R will still speak in favor of the left. At the cutoff value, R is indifferent between speaking in favor of either ideology, hence leading to a mixed strategy equilibrium.

Media designing a report which extol their own ideology even in the face of a contradicting event follows (Baum and Groeling, 2009). The current model formalizes the sufficiency conditions where events contradicting a media's ideology will bind it to speak closer to the truth. Media j with value of λ_j greater than threshold speaks closer to the true events and does not jeopardize with reader-assessments while the ones below the threshold advocates more towards ideology motive, thereby publishing stories contradicting the true event.

The third statement highlights the conditions for symmetric equilibrium. For the existence, it is necessary that the event must have no ideological underpinnings. The sufficiency factor is that both media should have identical preferences towards ideology.

Remark 1. ***Comparison of magnitude of editorial positions:*** *The class of events which strictly favors the left, $\theta_E \in [-1, 0)$, L chooses to locate closer to the event than R. Analogously, for events favoring the right, R chooses to locate closer to the event than L.*

This phenomenon is illustrated through table 1. Additionally, L and R locate symmetrically around zero when $\theta_E = 0$ (neutral event) and $\lambda_L = \lambda_R$ holds (shown in bold in table 1).

Remark 2. *Intuitively, the threshold value $\bar{\lambda_j}$ of λ_j is a measure of the extent to which L is willing to champion its ideology in presence of a contradicting reality.*

$(\lambda_L, \lambda_R) \setminus \theta_E$	-1	0	1
(0.1,0.1)	(-0.986,-0.346)	**(-0.417,0.417)**	(0.346,0.986)
(0.1,0.5)	(-0.988,-0.509)	(-0.387,0.279)	(0.344,0.908)
(0.1,0.9)	(-0.989,-0.722)	(-0.365,0.07)	(0.331,0.748)
(0.5,0.1)	(-0.907,-0.344)	(-0.279,0.389)	(0.508,0.988)
(0.5,0.5)	(-0.919,-0.507)	**(-0.258,0.258)**	(0.506,0.92)
(0.5,0.9)	(-0.93,-0.719)	(-0.242,0.064)	(0.497,0.789)
(0.9,0.1)	(-0.748,-0.331)	(-0.071,0.366)	(0.722,0.989)
(0.9,0.5)	(-0.789,-0.497)	(-0.065,0.242)	(0.72,0.93)
(0.9,0.9)	(-0.827,-0.713)	**(-0.06,0.06)**	(0.712,0.827)

Table 1: The first column shows that when true state totally favors the left, then L speaks closer to the truth than R for all values of λ_R (in blue). Symmetric results hold for R (in red). As λ_L increases, L locates itself closer to the median reader at 0. The only symmetric equilibrium occurs when $\theta_E = 0$ and $\lambda_L = \lambda_R$(shown in bold).

Remark 3. *The probability p_j with which media $j \in \{L, R\}$ randomizes between reporting the negative event and speaking in favor of own ideology is independent of all model parameters, but the nature of event at hand, i.e θ_E.*

Given that θ_E favours j's rival, it is common knowledge that its rival will speak in favor of its won ideology. When j has its trade-off between ideology and reader-evaluation equal to the cutoff $\bar{\lambda_j}$, the choice of location on either side of zero depends on the description or θ_E. j will be indifferent between speaking for either ideologies. It is later shown that the profit of j is the lowest at this cut-off value.

Proposition 2. *(i) There exists a reader in $[-1, 1]$ with ideology x_i who assigns identical assessments to news of L or R. x_i can be uniquely solved from the below identity when $\theta^*_L \neq -\theta^*_R$*[11]

$$(x_i + \theta_E)\Big(\frac{1}{\theta^*_L} + \frac{1}{\theta^*_R}\Big) = 4$$

(ii) If an incident supports the left, and media R's weight on reader assessment is high enough, then a fraction of leftist readers will prefer L's news over that of R even if the latter is closer to their ideology.

The first statement resonates the idea of Hotelling's linear city while the second statement goes against it. The latter reflects the idea that if the right media reports a pro-left event, then a fraction of leftist readers surrounding the location chosen by the right media would still prefer the left media news story.

[11]The outcome $\theta^*_L = -\theta^*_R$ is endogenously arrived iff $\lambda_L = \lambda_R = 1$ and are reporting a neutral event ($\theta_E = 0$). In this case, the median reader at 0 is indifferent between either outlets. We can exclude this case as λ_L and λ_R lies between $(0, 1)$.

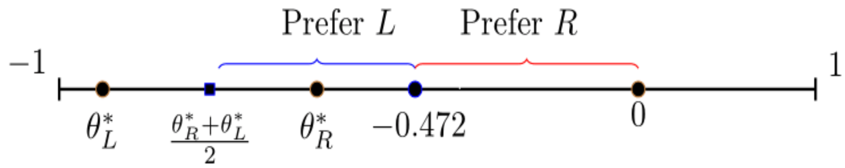

Figure 4: This illustrates the reporting of a extreme pro-left event ($\theta_E = -1$). The readers between $[\frac{\theta_L^*+\theta_R^*}{2}, -0.472)$ assigns better assessment to L's report than R even when L is located father away from them than R. However, leftist readers between $[-0.472, 0)$ provides better assessment to R's news than that of L, by the reasoning of Hotelling.

Corollary 1. *Readers strictly to the left of* $\frac{4\theta_L^*\theta_R^*}{\theta_L^*+\theta_R^*} + \theta_E$ *enjoys news from the left media while those to the right enjoy news of the right media.*

Corollary 2. *If L and R are reporting an incident which completely supports the left, $\theta_E = -1$, with $\lambda_L \rightarrow 1$ and $\lambda_R \rightarrow 0$ respectively, then a fraction of leftist readers who are closer to R's location choice than that of L assess news of L better than R.*

If media L is not motivated enough towards deriving ideology payoffs but competes with media R which is more ideologically motivated, then some relatively weaker leftist readers will prefer news about a pro-left issue from media R. These readers who are weakly attached to either ideology will discount their like-minded news source which will take a relatively indifferent stance. On the contrary, they will gain more satisfaction from receiving positive news about their ideology from an ideologically opposite media. A numerical example can be given to throw more clarity. When $\theta_E = -1$ (extreme pro-left event) and $(\lambda_L, \lambda_R) \rightarrow (0.998, 0.002)$ then $(\theta_L^*, \theta_R^*) \approxeq (-0.998, -0.633)$ which implies that leftist readers approximately between $(-0.472, 0)$ prefers news from media R than L. Readers to the right of this interval prefers R, while those to the left are more satisfied with L.

5.3 Choice of reporting neutrally

There occurs two broad scenarios where media j can report neutrally by locating closer to zero. First, when the true event is actually neutral and second, when the event is unfavourable to j's ideology. The former indicates truthful reporting, while the latter can be termed as 'indifferent reporting', a form of biased news reporting, where the media is reluctant to speak in favor of the rival ideology. However, as this model predicts from proposition 1, media does not want to sound indifferent even when faced with an ideologically 'bad' event.

For example, when the event favors the left (θ_E^1 in figure 5), then L does not position itself in the blue region. When L is more attached to its ideology ($\lambda_j \leq \hat{\lambda}_j$), it places itself on the left of the

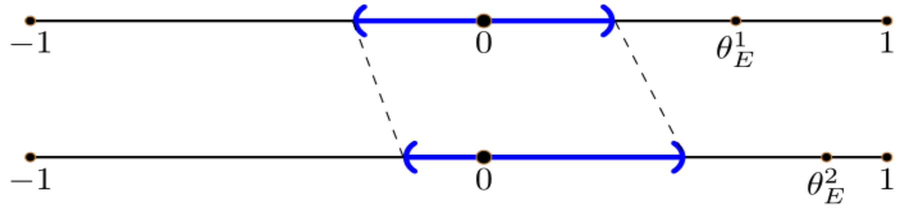

Figure 5: **Deviation from neutral reporting**: Suppose, readers are uniformly distributed and there occurs an event θ_E^1 which favors the right. Then L does not locate on the blue region. It reports on the left of this region (supporting the left) when λ_L lies below the cut-off $\bar{\lambda_L}$ (it is ideologically stronger) and reports on the right for values of λ_L above $\bar{\lambda_L}$ (it is more motivated towards reader-assessment). When the event favors the right more strongly,say θ_E^2, then this blue region shifts to the right.

blue interval. On the other side of this cutoff, L places itself in the territory of the rightist readers, on the right of the blue interval. In essence, L avoids a more indifferent location (around zero)

I observe that when the true event is neutral ($\theta = 0$), then it is strategically dominant strategy to bias news. I get a closed form solution of θ_j^* from the first order conditions.

Proposition 3. *(i) When $\theta_E = 0$, the equilibrium editorial choice of j is given by*

$$\theta_j^* = \frac{(1 - \lambda_j)}{(1 - \lambda_j) + \frac{c}{b + (\theta_{-j}^*)^2}}$$

(ii) Given any unfavourable event, media j either supports its own ideology or the opposite ideology conditional on the value of λ_j. However it never locates on a region surrounding zero which implies indifferent reporting.

The technical proof is in the appendix. The first statement is analogous to Osborne and Pitchik (1987) where the two firms locate at a distance of roughly 0.27 from either ends of $[0, 1]$ interval. Firms choose this by minimizing the consumer's transportation cost which in our model reflects the cost of reading a news story which is far away from a reader's ideology. Locating at the midpoint of the $[-1, 1]$ interval only increases the transportation costs of extreme readers

The implication of the second statement can be derived from figure 5. If there is a rise in inflation during the presidency of the left, then statement ii implies that if L is too partisan-motivated, then it will detach the effect of the left ideology with the rise in inflation by report that its unemployment reducing monetary policies are targeted to lower unemployment which comes at a cost of higher inflation or raise doubts in readers' minds about the possibility that the reported numbers as overestimated. Alternatively, if L is more motivated towards reader-assessments, then the coverage can come as a criticism of the policy which lead to higher inflation.

The above phenomenon was found in the way Fox news also covered ICE's decison of deporting international students during pandemic. The news story did not criticise the decisions but highlighted the dire impact it had on the lives of international students.[12] What appears is that media will speak (not strongly enough) in favor of its adversary instead of positioning itself near zero, which intuitively leads to a tendency to build better reader-assessment credibility even from opposite-minded readers.

5.4 Comparative Statics

I now consider how the parameters λ_L and λ_R affect the equilibrium choices of L and R respectively. For more clarity of the stated propositions, I study the effects of the equilibrium choices of media L. Analogous explanations will hold for a similar study of R's equilibrium choice. I also study the cross-over effect the rival media imposes on the equilibrium choices of the media firms (through the parameter b). Applying IFT to (5), I arrive at the following

$$\frac{d\theta_L^*}{d\lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25\theta_E^2 - 0.5\theta_E\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L\theta_E} \tag{6}$$

Let us take numerical values of exogenous parameters to better understand the comparative statics. I choose $b = 0.7$ and $c = 1.1$ and given $\lambda_L = \lambda_R = 0.1$, I get $(\theta_L^*, \theta_R^*) = (-0.417, 0.417)$ when $\theta_E = 0$. Incorporating in (8), I get,

$$\frac{d\theta_L^*}{d\lambda_L} = 0.27$$

As I will see later that this magnitude is greater than the comparative statics result from the monopoly model in section 6 where $\frac{d\theta_L^*}{d\lambda_L} = 0.2475$. Intuitively, given $\theta_E = 0$, when media L puts more weight on payoffs from readers, then it takes en editorial stance closer to the median reader.

I now conduct a similar comparative statics exercise with parameter b. This will allow us to measure the cross-effects of editorial choice of R on the choices of L and vice versa. Using IFT on equation (5) through parameter b gives us the following equality.

$$\frac{d\theta_L^*}{db} = \frac{\frac{-c\theta_L^3(1-\theta_L)}{(b+(\theta_R-\theta_E)^2)^2}}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L\theta_E} \tag{7}$$

As the weight on satisfying the average reader increases, both rival partisan media firms tries to place themselves near the median reader. The sign of the derivatives shows that the equilibrium editorial stance of L moves rightward towards 0 while the position of R moves leftward towards 0.

The nature of signs of the change in equilibrium level of slant depends on whether the events are themselves too strongly or too weakly biased. As before, $[\underline{\theta_E^L}, \bar{\theta_E^L}]$ depicts events which are

[12] A report by Fox5 Atlanta on July 8th 2020 titled "International students face uncertain future due to new ICE rule".

weakly biased (centered around 0) while its complement within $[-1,1]$ denote the events which are biased strongly enough to either ideology.

Proposition 4. *(i) If an event favors j's ideology, j's editorial choice moves closer to the median reader at 0 as λ_j increases. In other words,θ_L^* increases with λ_j, ($\frac{d\theta_j^*}{d\lambda_j} > 0$).*

(ii) For any unfavourable event, $\frac{d|\theta_j^*|}{d\lambda_j} < 0$ for all $\lambda_j \in (0, \bar{\lambda_j})$ and $\frac{d|\theta_j^*|}{d\lambda_j} > 0$ for all $\lambda_j \in (\bar{\lambda_j}, 1)$. At $\bar{\lambda_j}$, θ_j^* is discontinuous.

(iii) The impact of a more sophisticated reader pool reduces bias of L given any nature of event. However, the weights on ideology and reader-assessment of both media weakens or strengthens this impact.

(a.) When the event has no ideology ($\theta_E = 0$), the impact gains strength in the presence of a media R which is less focused on ideology motive and assigns greater weight on reader assessment.

(b.) If the event supports the ideology of media L, then the impact is greater in the presence of R whose motive is more driven towards ideology gains than reader-assessment.

Sub-part (i) points out that as j's attachment towards its own ideology falls, its comparative statics with respect to λ_j naturally segments the event space into two classes - events which are biased enough vis-a-vis the ones which are not. For the first class, media L's editorial choice increases (moves towards right on the ideology spectrum) when the weight on reader satisfaction increases. Analogously media R's response decreases and moves towards the left.

(ii) implies that θ_j^* is piecewise continuous. As stated in statement (ii) of proposition 1, at the advent of a ideologically negative event, media j's location strategy varies distinctly around a threshold value of λ_j. This variation is clearly suggested by the direction in change of θ_j^* on either side of the threshold. At the threshold, θ_j^* exhibits non-removable discontinuity of the first kind where $\theta_j^*(\bar{\lambda_j} + 0)$ and $\theta_j^*(\bar{\lambda_j}) - 0)$ exists but have different values. θ_j^* remains continuous for all other values of λ_j.

Higher value of b, implies lower reader sophistication, thereby a greater leeway to bias in favor of ideology. According to Ansolabehere, Behr and Iyengar (1993), more educated people will generally call upon alternative information before accepting a news story and that increases the likelihood of them positing a stronger counter-argument to a overtly biased news story. This argument augments the third statement. The effect of a more sophisticated reader-base on reducing bias of a particular media is affected by contemporaneous effects of the preference of its rival. When the event has no bearing on ideology, then the presence of a rival which prefers reader-assessement will lead to a reduction in bias. This is because, it would pay the media more to locate towards the median reader by the standard Hotelling argument.

To evaluate the effect of increasing λ_L on the equilibrium payoffs, I use envelope theorem. By the envelope theorem, the effect of any parameter on the maximum value function is entirely the direct effect of the parameter on the maximum value function. The maximum value function V_j is calculated by substituting θ_j^* in the payoff functions of media j.

Proposition 5. ***Responsiveness of maximum value function***

(i) Suppose the event is neutral ($\theta_E = 0$), then the maximum value function decreases as the weight on reader ratings are increased, $\frac{dV_j}{d\lambda_j} < 0$.

(ii) Suppose the event is not neutral ($\theta_E \in [-1, 0) \cup [-1, 1]/\{0\}$), then the maximum value function is U-shaped as λ_j as increased.

The technical proof is in the appendix. What is implied by this is the following. Given the reader pool is balanced and $\theta_E = 0$, the first term of media j's profit function is zero (see equation 2). Then profit in equilibrium will always be enhanced when $\lambda_L \to 0$, or media j is more ideology-motivated.

Intuitively, if the event is neutral, then an average reader has zero bias (balanced reader pool) and he will tune in to the partisan channels to learn about potential ideological subtleties. Hence, placing more weight on ideology brings in higher rewards for the media. Alternatively, placing weight of reader assessment and providing a neutral report only leads to worse experience of like-minded readers. This resonates with the location choice model of Osborne and Pitchik (1987) where firms does not choose the midpoint of the linear city economy (of unit length) but at roughly at points 0.25 and 0.75.

The second statement means that while reporting a story which is not neutral (the story either supports or attacks the ideology of j), higher equilibrium profits are achieved when media either focuses on ideology or on reader ratings. Equilibrium profits are compromised if j wants to produce a report by balance both the factors. Hence, higher profits are realized at the extreme values of λ_j.

By continuity of the maximum value function, then there exists a threshold where the media experiences the lowest equilibrium profit. This is the exact threshold which reflects the desperation of a media to support its ideology even when the true event stands in contradiction. Following proposition 1, this threshold is denoted by $\bar{\lambda_j}$ and $\frac{\partial V_j}{\partial \lambda_j}$ vanishes at $\bar{\lambda_j}$.

Remark 4. *At the threshold value, media experiences greater equilibrium losses which primarily stems from poorer reader ratings.*

Desperately supporting its ideology can besmirch j's image to a certain mass of readers who will doubt j's credibility. This also intuitively connects to the experimental findings of Baum and Groeling (2009) where media often engage in such risky editorial decisions. As I will see from the comparative statics section below that this type of reporting comes at a high cost. At this threshold, media actually experiences the highest equilibrium loss.

6 Equilibrium in a left or a right majority reader pool

Until now, this paper has dealt with the case where the share of leftist and rightist readers in the economy readers. I now study the equilibrium strategies of L and R which publishes reports to a distribution of readers who are either left or right-leaning.

I assume that media has perfect knowledge about the mass of N readers who lie in their territory on $[-1, 1]$.

The aggregated α_{ij} across N readers towards media $j \in \{L, R\}$ in equation (3) is :

$$\alpha_j^* = \sum_{i=1}^{N} \frac{x_i + \theta_E}{2\theta_j^*}.\frac{1}{N} = \frac{\theta_E}{2\theta_j^*}$$

The political neutrality or balance between the share of leftist and rightist readers was formalized by $\sum_{i=1}^{N} x_i = 0$, thereby arriving at the above result.

Relaxing the condition in this section entails $\sum_{i=1}^{N} x_i \neq 0$. Equation (3) then becomes,

$$\alpha_L^* = \sum_{i=1}^{N} \frac{x_i + \theta_E}{2\theta_L^*}.\frac{1}{N} = \frac{\kappa + \theta_E}{2\theta_L^*}, \kappa \neq 0 \tag{8}$$

I now have two possible scenarios:

1. Majority of readers are rightist: $\frac{1}{N}\sum_{i=1}^{N} x_i = \kappa$ and $0 < \kappa < 1$.
2. Majority of readers are leftist: $\frac{1}{N}\sum_{i=1}^{N} x_i = -\kappa$ and $0 < \kappa < 1$

First order condition for media L now becomes:

$$\begin{aligned}\frac{d\Pi_L}{d\theta_L} = \theta_L^4\left[(1-\lambda_L) + \frac{c}{(b+(\theta_R-\theta_E)^2)}\right] + \theta_L^3\left[(1-\lambda_L) - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}\right] \\ + 0.5\lambda_L(\theta_E+\kappa)\theta_L - 0.25(\theta_E+\kappa)^2\lambda_L = 0\end{aligned} \tag{9}$$

First order condition for R is

$$\begin{aligned}\frac{d\Pi_R}{d\theta_R} = \theta_R^4\left[(1-\lambda_R) + \frac{c}{(b+(\theta_L-\theta_E)^2)}\right] - \theta_R^3\left[(1-\lambda_R) + \frac{c\theta_E}{b+(\theta_L-\theta_E)^2}\right] \\ + 0.5\lambda_R(\theta_E+\kappa)\theta_R - 0.25(\theta_E+\kappa)^2\lambda_L = 0\end{aligned} \tag{10}$$

Figure 6: If there is a leftist majority of κ_L, then L will always support its own ideology for events in the cyan region. For events in the blue region, which are biased more strongly than the ones in cyan, L will support its leftist (rightist) ideology for $\lambda_L < \tilde{\lambda_L}$ ($\lambda_L > \tilde{\lambda_L}$). At the threshold, L is indifferent. One can interpret the orange and red intervals for media R in an analogous fashion.

In equilibrium, the strategy pair (θ_L^*, θ_R^*) solves (9) and (10). The next proposition presents how the presence of a dominant leftist-reader base emancipates media L while restraining R and vice-versa. We draw in comparisons with equilibrium editorial choices with balanced reader base in $Proposition 1$ and also combine the comparative statics due to changes in trade-off between ideology and ratings. The comparative statics results follows from applying the implicit function theorem on (9) and (10). His is similar to what we have done before.

Proposition 6. *Suppose that media j has majority share of like-minded readers equal to κ_j.*

(i) ***Pure strategy equilibrium:*** *exists where j favor its own ideology alongside an added interval of events of length $\tilde{\theta_E}(\kappa_j)$ which supports the rival ideology. When the support for their rival goes above $|\tilde{\theta}|$, then j supports its rival. Comparative statics on θ_j^* due to λ_j is continuous, i.e, $\frac{d\theta_j^*}{d\lambda_j} > 0$.*

(ii) ***Mixed strategy equilibrium:*** *exists for For class of events lying beyond $\tilde{\theta}_j\kappa_j$, a mixed strategy equilibrium exists at $\lambda_j = \tilde{\lambda_j}$ where j is indifferent between supporting its own or rival ideology. Unique pure strategy equilibrium exists for other values of λ_j, where j supports its own (rival) ideology below (above) the threshold $\tilde{\lambda_j}$.*

This is easier to explain using figure 6. Suppose leftist readers form a majority share of κ_L. Given this, any event lying to the left of zero will be reported by media L with more intensity for all values of λ_L (this is consistent with *proposition 1*). In the current proposition, the dominance of like-minded readers by a magnitude, say κ_L, offer L a leverage to bias events which also lie to the right of zero, (denoted by the cyan interval $(0, \tilde{\theta_E}(\kappa_L))$ in figure 6). However, κ_L does not give L the liberty to unconditionally bias news for topics lying to the right of the cyan region. So for events in the blue interval, biasing information becomes conditional on λ_L (L will bias in favor of left (right) below a threshold value $\tilde{\lambda_L}$ of λ_L, from proposition 1). For events lying to the right of the blue interval (topics more strongly favoring the right), L speaks in favor of the right for all values of λ_L. Symmetric interpretation with regard to media R's strategy can be given for events lying in the orange and red intervals.

Corollary 3. *Comparing the threshold levels at which mixed equilibrium is supported between situations of balanced and unbalanced reader population, we find that*

$$\bar{\lambda_L} < \tilde{\lambda_L}$$

To elicit some important characteristics of media behavior, we first assume the parameters signifying the majority share and the cost of biasing through cross-over effect to be below some cutoff such that $|\kappa| < \hat{\kappa}$ and $b \geq \hat{b}$. This guarantees that the editorial choice are not shackled too much either by the extent of biased readers or by a very high cost of bias.

Remark 5. *(i) The magnitude of movement of θ_L^* is dictated by the gross effect of the absolute values of $\frac{d\theta_L^*}{d\lambda_L}$ and $-\frac{d\theta_L^*}{d\kappa}$. θ_L^* moves towards (away from) 0 if the net effect is positive (negative).*

(ii) Analogously, the magnitude of movement of θ_R^* is dictated by the net effect of $\frac{d\theta_R^*}{d\lambda_R}$ and $\frac{d\theta_R^*}{d\kappa}$. θ_R^* moves towards (away from) 0 if the net effect is positive (negative).

When λ_L and λ_R increases, then media L and R respectively concentrates more on average reader, hence moves towards the mean reader. As κ decreases (readers are majorly left-biased), it gives L more leeway to position itself more extremely towards the left. Symmetrically when κ increases (readers are majorly right-biased), then R has more freedom to bias in favor of the right.

We now examine the features of the maximum value function and draw in comparisons with the statements in proposition 3.

In essence, higher profit is enjoyed by media which are either more ideologically extreme (close to -1 or 1) or mainly care only about reader ratings (close to 0). The sequence of thresholds $\{\bar{\lambda_L}\}$ where equilibrium profit equals zero, originates from a point which is closer to the optimal ideology at -1 than in a situation when reader pool was balanced (Proposition 3). This implies that, in an economy dominated by leftist readers, by compromising with ideology (by increasing λ_L away from some $\epsilon \to 0$), L gets penalized in terms of media ratings.

7 Model with 3 media outlets

We expand the previous analysis by adding one more firm on the ideology axis. We denote this firm by Q which has an ideological bliss point at $\tilde{q} \in (-1, 1)$. The remaining features of the model comprising the readers and the media outlets L and R carries on unchanged in this section. This exercise is expected to reveal how more competition among the media outlets affect the equilibrium level of bias.

The corresponding normal form game of this three firm model is defined as
$\Gamma_T = [I, \{S_i\}, \{S_L\}, \{S_R\}, \{S_Q\}, \{u_i(.)\}, \{\Pi_L(.)\}, \{\Pi_R(.), \{\Pi_Q(.)\}]$. I denotes the player set

comprising of media L, R and Q and reader $i \in \{1,..,n\}$. u_i is the utility of reader i from reading news and Π_L,Π_R and Π_Q denotes the profits of media L. R and Q. Thereby the strategy profile constituting the SPNE is characterized as $s^* = (\theta_L^*, \theta_R^*, \theta_Q^*, \alpha_{iL}^*(\theta_L^*), \alpha_{iR}^*(\theta_R^*), \alpha_{iQ}^*(\theta_Q^*)) \quad \forall \quad i = \{1,..,N\}$.

7.1 Utility Maximization of reader

We will inherit equation (1) with one more media firm Q such that for $j \in \{L, R, Q\}$, utility of any reader i is given by

$$U_i(\alpha_{ij}|\theta_j, \theta_E) = -(\alpha_{ij}\theta_j - x_i)^2 - (\alpha_{ij}\theta_j - \theta_E)^2$$

7.2 Backward Induction by Media

With three firms, the payoff function takes a slightly revised form where the natire of cost function gets updated to account for the bias of the third firm. In the following three equations, we layout the payoffs of media $j \in \{L, R, Q\}$.

$$\Pi_L(\theta_L, \theta_R, \theta_Q) = -\lambda_L.(\alpha_L^* - 1)^2 - (1-\lambda_L)(\theta_L + 1)^2 - \frac{c(\theta_L - \theta_E)^2}{b + (\theta_R - \theta_E)^2 + (\theta_Q - \theta_E)^2} \quad (11)$$

$$\Pi_R(\theta_R, \theta_L, \theta_Q) = -\lambda_R.(\alpha_R^* - 1)^2 - (1-\lambda_R)(\theta_R - 1)^2 - \frac{c(\theta_R - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_Q - \theta_E)^2} \quad (12)$$

$$\Pi_Q(\theta_Q, \theta_L, \theta_R) = -\lambda_Q.(\alpha_Q^* - 1)^2 - (1-\lambda_Q)(\theta_Q - \tilde{q})^2 - \frac{c(\theta_Q - \theta_E)^2}{b + (\theta_L - \theta_E)^2 + (\theta_R - \theta_E)^2} \quad (13)$$

The only difference between L's (R's) payoff function from previous section lies in the cost function which now takes account for the bias of the the third media house Q.

Definition 2. *Nash Equilibrium of this game Γ_T is a triple $(\theta_L^*, \theta_R^*, \theta_Q^*)$ of editorial choices for which θ_j^* is a best response to θ_{-j}^* where $j \in \{L, R, Q\}$*

The below table shows a numerical depiction of the equilibrium choices of L and R with the entry of a new media with two respective ideology bliss points- -0.5 and -0.75 and for two values of $\lambda_Q = \{0.1, 0.5\}$.

(λ_L, λ_R) | λ_Q	0.1	0.5
(0.1,0.1)	(-0.436,0.436,-0.23)	(-0.426,0.426,-0.162)
(0.1,0.5)	(-0.4,0.293,-0.218)	(-0.396,0.28,-0.15)
(0.5,0.5)	(-0.27,0.27,-0.20)	(-0.264,0.264,-0.138)

Table 2: Equilibrium editorial position of media L , R and Q $(\theta_L^*, \theta_R^*, \theta_Q^*)$ when $\theta_E = 0$ and Q is located at -0.5. For comparison purposes, we have highlighted $L(R)$'s choices in blue (red) for $\lambda_Q = 0.1$.Editorial choices are more extreme with the new biased media Q from the duopoly model in Table 1.

(λ_L, λ_R) | λ_Q	0.1	0.5
(0.1,0.1)	(-0.46,0.46,-0.36)	(-0.436,0.436,-0.247)
(0.1,0.5)	(-0.426,0.31,-0.33)	(-0.41,0.294,-0.226)
(0.5,0.5)	(-0.285,0.285,-0.31)	(-0.27,0.27,-0.20)

Table 3: Equilibrium editorial position of media L, R and Q $(\theta_L^*, \theta_R^*, \theta_Q^*)$ when $\theta_E = 0$ and Q is located at -0.75.For comparison purposes, we have highlighted $L(R)$'s choices in blue (red) for $\lambda_Q = 0.1$. Editorial choices are not only more extreme from the duopoly model, but also from Table 2 where Q is relatively less biased.

Taking into account of the nature of profit functions in $16, 17$ and 18, we have the following corollary.

Remark 6. *Given $\theta_E = 0$, the equilibrium editorial choices of L and R become more biased with the entry of a biased third firm Q. If Q is unbiased or positioned at 0, then it has no effect on the equilibrium editorial choices of L and R.*

If media is covering a story about a neutral event, then the entry of a new firm which is biased, increases the absolute levels of slants in both L and R. To get more clarity, one can compare the numbers of the editorial choices of L (R) highlighted in blue (red) across Tables 1, 2 and 3. The proof of the second part is straightforward and entails that the presence of the unbiased media is unable to cater to the ideological beliefs of readers along the ideological spectrum- thereby, biased media firms stay persistent in their prior editorial choices and refuses to decrease their slant.

8 Welfare analysis and policy prescription

In this spatial linear city model of product placement, where firms with different biases compete to serve news to readers who also differ, truth revelation comes with a trade-off. The primary objective of welfare improvement lies not only when media outlets reports accurately, but also when the readers perceive the news in light of the facts underlying the issue and not let their ideology override their judgement of those facts. This trade-off gets worse in presence of reader polarization and reader population as this section as this analysis will show. I first layout how reader surplus varies with more or less polarization, followed by a general description of how media profits and reader surplus varies with reader polarization and the issue to be reported. Finally I suggest policy prescription which will lead towards truth revelation alongside moderating the trade-off of the ideology effect which can dictate a reader's judgement.

8.1 Reader surplus

Reader surplus from a news story is represented by the gap between the value of assessment and 1 (this follows from Lemma 1 which shows the first best assessment value equals 1). This reflects the idea of deriving consumer surplus in by subtracting market price from the reservation price of a consumer. *Lemma 1* therefore entails that the first best assessment is akin to the reservation weight of a reader.

Following from *Equation 2*,

$$\alpha^*_{ij} = \frac{x_i + \theta_E}{2\theta^*_j} = \frac{x_i + \theta_E}{\theta^*_j + \theta^*_j} \tag{14}$$

Utility of i from reading a report of j is given as

$$U_i(\alpha^*_{ij}, \theta^*_j|\theta_E) = -(\alpha^*_{ij}\theta^*_j - x_i)^2 - (\alpha^*_{ij}\theta^*_j - \theta_E)^2$$

Therefore utility loss of i from a news to which i attaches a weight of $\hat{\alpha_j} \neq 1$ is

$$\Delta U_{ij} = U_i|_{\alpha^*_{ij}=\hat{\alpha_j}} - U_i|_{\alpha^*_{ij}=1} = 4(\theta^*_j - \frac{x_i + \theta_E}{2})^2 \tag{15}$$

Given 15, any reader i who faces zero utility loss is characterized by $x_i = 2\theta^*_j - \theta_E$. The total utility loss across all N readers due to media j is then calculated by summing the individual utility losses across N readers, denoted as

$$\Delta U_j = \sum_{i=1}^{N} \Delta U_{ij} = 4N((\theta^*_j)^2 - \theta_E\theta^*_j) + N\theta_E^2 - 2\theta_E \sum_i x_i + \sum_i x_i^2$$

or,

$$\Delta U_j = 4N((\theta^*_j)^2 - \theta_E\theta^*_j) + N\theta_E^2 - 2\theta_E\kappa + \sum_i x_i^2 \tag{16}$$

The above equation is intuitive and brings out the avenues where utility of readers decreases in the economy. The first term within parenthesis resembles the level of loss imposed on readers when media reports the true event. This term increases if media speaks overtly opposite to the truth ($\theta_E.\theta_j < 0$). The third term signifies whether the the event is favourable to the majority readers ($\theta_E.\kappa > 0$). It is quite evident that any policy interventions that can be implemented must be focused on taxing media firms to report closer to the truth, hence lowering the first term. Alternatively, policies to enhance readers to weigh the true event more than their ideology affiliations can lead to welfare improvement when both media reports closer to true event. The remaining terms are exogenous and no welfare improving policy can target to mitigate this loss.

I illustrate some simple examples with different reader demography which illustrates that policies will have a bite only relating to the first term. The effect of demography will either augment or impede the goal of any policy. I consider a fixed event and elaborate the loss reader will face from either L or R.

Example 1. ***Balanced readership with high polarization****: Consider an economy with 4 readers such that two are located at -0.5 and two at 0.5. The reader pool is balanced as both readers on either side of zero neutralize each other. However this population has variance of 1 ($\sum_i x_i^2 = 1$). Suppose now there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to media L and R is given by*

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 5$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 5$$

Example 2. ***Balanced readership with low polarization****: Consider an economy with 4 readers located at -0.5, -0.25, 0.25 and 0.5. The reader pool is still balanced as example 1. However the variance term is now lower, $\sum_i x_i^2 = 0.625$. Suppose there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to any media L and R is given by*

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 4.625$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 4.625$$

Example 3. ***Readership biased towards the event****: Now assume that these 4 readers are at -0.5, -0.30, -0.1 and 0.5. The reader pool now is left biased by -0.4 variance of 0.6. Suppose there occurs an event totally favorable to the left, $\theta_E = -1$. The loss in reader surplus due to any media L and R is given by*

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 3.8$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 3.8$$

Example 4. ***Readership biased against the event***: *Consider now that the* 4 *readers are at* -0.5*,* 0.30*,* 0.1 *and* 0.5*. The reader pool now is right biased by* 0.4 *variance of* 0.6*. Suppose there occurs an event totally favorable to the left,* $\theta_E = -1$*. The loss in reader surplus due to any media* L *and* R *is given by*

$$\Delta U_L = 16((\theta_L^*)^2 + \theta_L^*) + 5.4$$

$$\Delta U_R = 16((\theta_R^*)^2 + \theta_R^*) + 5.4$$

The constant terms resembles the demography effects on reducing consumer surplus on the entire economy due to media L and R. The exogenous effect of having high polarization comparative to low polarization can be estimated by the difference of the constant terms ($5 - 4.625 = 0.375$) in example 1 and 2. Analogously, the differences in the constant terms in examples 3 and 4 amounting to 1.6 ($5.4 - 3.8$) shows the exogenous effect on reader welfare due when the event stands contradictory to the majority's beliefs.

8.2 Media payoffs

The profit of any media j with ideological bliss point at $\dot{\theta}_j \in [-1, 1]$ is given by

$$\Pi_j(\theta_j, \theta_{-j}) = -\lambda_j.(\alpha_j^* - 1)^2 - (1 - \lambda_j)(\theta_j - \dot{\theta}_j)^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2}$$

Applying the characterization of α_j^* in *Lemma 1*, the profit of media j is

$$\Pi_j(\theta_j, \theta_{-j}) = -\lambda_j.\left(\frac{\kappa + \theta_E}{2\theta_j^*} - 1\right)^2 - (1 - \lambda_j)(\theta_j - \dot{\theta}_j)^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2} \qquad (17)$$

Equilibrium welfare (henceforth, welfare) denoted by W comprises of media j's payoff and the loss in utility faced by N readers. It then becomes evident that absolute number of readers N, their share net of ideology κ and the spread of readers from 0 denoted by $\sum_i x_i^2$ affect welfare, however to different extents.

$$W_j(\theta_j, \theta_{-j}|\theta_E, \kappa, N, c, b) = \Delta U_j(\theta_E, \kappa, N) + \Pi_j(\theta_j, \theta_{-j}|\theta_E, \kappa, c, b)$$

Remark 7. *(i) If the number of readers* N *increases, then welfare changes by* $4((\theta_j^*)^2 - \theta_E\theta_j^*) + \theta_E^2$*.*
(ii) If the share of readers, κ *increases, then welfare changes by* $-\frac{\lambda_j\kappa}{(\theta_j^*)^2} + \frac{\lambda_j}{\theta_j^*} - 2\theta_E$
(iii) Welfare decreases at a unit rate with the rise in variance of readers on the ideology axis.

The first two statements implies that readers face some utility loss from reading news about an event which goes against their ideological orientation. As the number of readers increases, it gets more challenging to satisfy everyone. Similarly, if the reader pool is skewed to the left or right, then welfare increases or decreases depending on the nature of event.

The final statement is relatively unambiguous about the nature of welfare change. It suggests that as the spread of readers rises, it gets more tough to satisfy them irrespective of the nature of event or the level of heterogeneity among readers.

8.3 Policy recommendation

By expanding the role of the honest media, truth telling can be better sustained such that news consumers start to accept the truth without being clouted by their ideology beliefs. To achieve this, I suggest that government spending can be targeted to develop the honest media in taking up an educative role while releasing the factual report. The aim of such an initiative lies in making readers assign greater weight on the true information than their ideology.

The degree of command of the media in making readers process the factual report is parameterized by $\beta \in (1, 2]$. The utility of readers from reading news reports of media L and R is represented by

$$U_{ij} = -(\hat{\alpha_{ij}}\theta_j - (2-\beta).x_i)^2 - (\hat{\alpha_{ij}}\theta_j - \beta.\theta_E)^2 \tag{18}$$

As shown above, any reader i while reading news of media j assigns a weight of β to the factual information which is strictly greater than the weight on ideologyx_i. The first order condition leads to

$$\hat{\alpha_{ij}}^* = \frac{(2-\beta).x_i + \beta.\theta_E}{2\hat{\theta}_j^*} \tag{19}$$

Media j's profit function is given by

$$\Pi_j(\hat{\theta}_j, \hat{\theta_{-j}}) = -\lambda_j.\left(\hat{\alpha_{ij}}^* - 1\right)^2 - (1-\lambda_j)(\theta_j - \dot{\theta}_j)^2 - \frac{c(\theta_j - \theta_E)^2}{b + \sum_{-j}(\theta_{-j} - \theta_E)^2} \tag{20}$$

The choice variable of $\hat{\theta}_j$ is now a function of β alongside the previous parameter used in the baseline model. To assess the merits of this policy intervention, I discuss the sufficiency conditions when the overall reader welfare in the economy. These conditions depend on how media j chooses to cover a topic considering the nature of reader polarization and given the value of λ_j.

Proposition 7. *(i). Suppose the event is neutral ($\theta_E = 0$).*

a. Then overall reader welfare is improved for any $\beta > 0$ if reader pool is balanced.

a. If the reader pool is biased, then reader utility is enhanced if the following is satisfied

$$\frac{\sum_i x_i^2}{4N}\Big(\frac{2-\beta}{\hat{\theta}_j^*} + \frac{1}{\theta_j^*}\Big) < \kappa$$

(ii). Suppose the event supports the left completely [13]*. Then the following conditions must hold for welfare improvement of readers.*

a. If the reader pool is also polarized completely to the left, then $\frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} > \beta$ *must be satisfied.*

b. If the reader pool is polarized completely to the right, then $2-\beta < \frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} < \beta(2-\beta)$ *must hold.*

c. If the reader pool is balanced, then $2-\beta < \frac{(\hat{\theta}_j^*)^2}{(\theta_j^*)^2} < \beta$*.*

See appendix section 10.9 for technical proof. In terms of intuition, when the topic is free of ideology, then strongly ideology oriented media will try to deviate and bias the news away from the point zero. If prior to media E undertaking such educative role, media j takes position θ_j^*, then ex-post E taking such a role media j moves towards zero by taking $\hat{\theta}_j^*$ which implies that $\frac{\hat{\theta}_j^*}{\theta_j^*} < 1$. The policy by media E generates a reporting interval where reader welfare will be enhanced if media j chooses to report within it. For higher values of β, such intervals are wider which suggests higher chances of welfare improvement.

In sub part (ii), when the event is itself favourable to the left, then welfare improvement becomes either redundant (implied by stronger the condition in a, when majority readers also share similar ideology beliefs with the topic) or challenging (when majority readers themselves hold beliefs opposite to the event, as in b). In b, when the topic is absolutely pro-left, then this condition is satisfied when R's motive is to maximize ideology payoffs ($\lambda_R \to 0$)

9 Concluding comments

The model presents a new way to understand how partisan media firms bias news of various topics if it accounts for its ideology and also how readers perceive news. The main trade-off that has been discussed is that ideological readers are reluctant to accept factual information. For example, leftist consumers would refuse to accept an factual academic report that criticizes its government's monetary policies which was unable to control inflation. If the leftist media L_1 is too ideology motivated, it will defend the policy by highlighting its power towards lowering unemployment. If another leftist media L_2 is not too blinded by ideology payoffs, then it might accept the critic. Now, how would the leftist and the rightist consumers perceive these reports? It is seen that a fraction of

[13] Exactly symmetric results will hold if the topic supports the right.

rightist readers would prefer the news of L_2, hence motivating more accurate reporting. However, more extreme rightist consumers would adhere to its like-minded media R and be more satisfied with its reporting.

In this set-up, policy measures should be designed which make factual information more receptive to the entire reader populace. I suggest that the exogenous media can take up an educative role by providing factual information in a manner such that the intrinsic facts get primary attention to the readers, thereby not letting their ideology override their judgement of a topic. Ideology worsens societal divide which raises disagreement regarding societal issues like proscribing abortion, anti-immigration attitudes [14] which according to the present analysis can be assuaged when people's reception of news is guided strongly by the facts of the matter. Simply increasing media competition by introducing less partisan media might lead to more accurate information provision, but its effect on reader perception remains ambiguous.

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10 Appendix

10.1 Example of news bias from mainstream news

As an example of how two opposing media can publish reports which evoke supremacy of their own ideology, I present the stories of CNN and Fox while covering the hike in tariff rates on Chinese goods by the Trump administration.

*CNN: **The US just raised tariffs on Chinese goods. China says it will hit back**: The United States has escalated its trade war with China, hiking tariffs on $200 billion worth of Chinese exports hours after trade talks held in Washington failed to produce a breakthrough. Tariffs on the targeted exports increased from 10% to 25% at 12:01 a.m. ET on Friday, prompting a swift rebuke from Beijing...“The tariff increase inflicts significant harm on US industry, farmers and consumers,” said Jacob Parker, vice president of the US-China Business Council...*

*Fox News: **Trump absolutely right to slap new tariffs on China**:President Trump on Sunday announced additional incoming tariffs on China, reminding Beijing that its days of negotiating* *with weak counterparts are over, at least as far as it concerns the United States. While Trump's move may cause short-term stock market turbulence, it's great news for U.S. national security and our economy over the longer term.*

As can be inferred, CNN's report is a blatant criticism of the policy and predicts a backlash from China while Fox News champions Trump for being aggressive with China, and hopes that this will instill renewed resilience on the part of the United States. Likewise, for any other incident, media will bias news bearing in mind its partisan interests and making its reader's happy.

10.2 Existence of roots of best response functions

Strum's theorem allows us to find the number of real distinct roots of each best response (BR) of L and R. This from Worth(2005) and helps us determine the number of real distinct roots within the interval $[-1, 1]$ for any given θ_E, λ_L and λ_R. This exercise allows us to know whether each of these equations have a real zero within $[-1, 1]$. The Nash equilibrium choices of θ_L and θ_R is then determined at the intersection of each of these BR.

We denote BR of L and R below as

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4\left[(1-\lambda_L) + \frac{c}{(b+(\theta_R-\theta_E)^2)}\right] + \theta_L^3\left[(1-\lambda_L) - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}\right] \\ + 0.5\lambda_L\theta_E\theta_L - 0.25\theta_E^2\lambda_L = 0 \tag{21}$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4\left[(1-\lambda_R) + \frac{c}{(b+(\theta_L-\theta_E)^2)}\right] - \theta_R^3\left[(1-\lambda_R) + \frac{c\theta_E}{b+(\theta_L-\theta_E)^2}\right] \\ + 0.5\lambda_R\theta_E\theta_R - 0.25\theta_E^2\lambda_R = 0 \tag{22}$$

Definition 3. *Strum's sequence: The Strum sequence for a univariate polynomial $f(x)$, is a sequence f_0, f_1, f_2... such that*
$f_0 = f$
$f_1 = f'$
$f_{i+1} = -rem(f_{i-1}, f_i)$ *where* $rem(f_{i-1}, f_i)$ *is the remainder when* f_{i-1} *is divided by* f_i*.*

Definition 4. *Strum's Theorem: - Let $f(x)$ be a polynomial of positive degree with real coefficients* *and let* $\{f_0(x) = f(x), f_1(x) = f'(x), f_2(x), ..., f_s(x)\}$ *be the standard sequence for* $f(x)$*. Assume* $[a, b]$ *is an interval such that* $f(a) \neq 0 \neq f(b)$*. Then the number of distinct real roots of* $f(x)$ *in* (a, b) *is* $V(a) - V(b)$ *where* $V(c)$ *denotes the number of variations in sign of the Strum's sequence* $\{f_0(c), f_1(c), ..., f_s(c)\}$

10.3 Proof of proposition 3

For sub part 1, I can simply substitute $\theta_E = 0$ in the above equations 21 and 22 to get

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4\left[(1-\lambda_L) + \frac{c}{(b+(\theta_R)^2)}\right] + \theta_L^3\left[(1-\lambda_L)\right] = 0$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4\left[(1-\lambda_R) + \frac{c}{(b+(\theta_L)^2)}\right] - \theta_R^3\left[(1-\lambda_R)\right] = 0$$

The above system leads to

$$\theta_L^* = \frac{(1-\lambda_L)}{(1-\lambda_L) + \frac{c}{b+(\theta_R^*)^2}}$$

$$\theta_R^* = \frac{(1-\lambda_R)}{(1-\lambda_R) + \frac{c}{b+(\theta_L^*)^2}}$$

For the second sub part, I again use equations 20 and 21 and substitute $\theta_E = 1$, when the event is extreme pro-right. Symmetric outcomes emerge when $\theta_E = -1$.

$$g(\theta_L) = \frac{d\Pi_L}{d\theta_L} = \theta_L^4\left[(1-\lambda_L) + \frac{c}{(b+(\theta_R-1)^2)}\right] + \theta_L^3\left[(1-\lambda_L) - \frac{c}{b+(\theta_R-1)^2}\right]$$

$$+ 0.5\lambda_L\theta_L - 0.25\lambda_L = 0$$

$$g(\theta_R) = \frac{d\Pi_R}{d\theta_R} = \theta_R^4\left[(1-\lambda_R) + \frac{c}{(b+(\theta_L-1)^2)}\right] - \theta_R^3\left[(1-\lambda_R) + \frac{c}{b+(\theta_L-1)^2}\right]$$

$$+ 0.5\lambda_R\theta_R - 0.25\lambda_R = 0$$

Since it is nearly impossible to derive a closed form solution of (θ_L^*, θ_R^*), I resort to solutions based on heuristics to give a suggestive solution about where the optimal values will lie. First I provide a closed-form solution of λ_L in presence of $\theta_E = 1$ and $\lambda_L = 0$. This is as follows,

$$\theta_L^* = -\frac{1 - \frac{c}{b+(\theta_R^*-1)^2}}{1 + \frac{c}{b+(\theta_R^*-1)^2}}$$

I now need to prove that θ_L^* is sufficiently away from zero and is positive. Now throughout the model, I have assumed $c = 1.1$ and $b = 0.7$. Then θ_L^* is positive iff $(\theta_R^* - 1)^2 < 0.4$. This implies that R locates between $(0.8, 1)$ in equilibrium. If R is motivated towards ideology more strongly, it will report very close to 1 and θ_L^* will be strictly positive. However, when R has almost no ideological motivation, R can place itself a bit away from zero [15]during which θ_L^* will be negative. This happens for $\lambda_L = 0$. So when λ_L is increased beyond zero, then the above inequality becomes less binding and is more easily satisfied.

If one refers to assumption 1 that $b > b'$, then it is reasonable to infer that with a lower value of b, the chances of truthful reporting increases which entails that when L has to report a pro-right event like the one discussed, it will locate farther away from zero towards that event, thereby refraining from indifferent reporting.

A more general way of presenting the conditions when media will refrain from locating near zero is by the following method. I first assume $\lambda_L = 0$ and incorporate it to 20 and 21 to get,

$$g(\theta_L) = \theta_L^4 \left[1 + \frac{c}{(b + (\theta_R - \theta_E)^2)}\right] + \theta_L^3 \left[1 - \frac{c\theta_E}{b + (\theta_R - \theta_E)^2}\right] = 0$$

This gives

$$\theta_L^* = -\frac{1 - \frac{c\theta_E}{b+(\theta_R^*-\theta_E)^2}}{1 + \frac{c}{b+(\theta_R^*-\theta_E)^2}}$$

As θ_E increases beyond zero, it raises the value of $c\theta_E$ which leads to a positive value of θ_L^*. Hence, with a more pro-right topic to cover, L will choose to locate at a point which is farther right away from zero. This holds for $\lambda_L = 0$. Hence for $\lambda_L > 0$ (no matter how small), this shift will be of greater magnitude.

Hence, it is proved that L will refrain from taking an indifferent stance while covering a pro-right event.

[15]One can refer to table 4 below of the monopoly model to see how any media j reports when λ_j approaches the value 1.

10.4 Proof of proposition 5

By envelope theorem, the effect of a change in the maximum value function is equal to the direct effect of the parameters. We differentiate the profit function of L in equation 4 at equilibrium editorial stance of L. This will also hold true for media R.

$$\frac{dV_L}{d\lambda_L^*} = -(\alpha_L^* - 1)^2 + (\theta_L^* + 1)^2$$

Upon expanding,

$$\frac{dV_L}{d\lambda_L^*} = \frac{\theta_E}{\theta_L} - \left(\frac{\theta_E}{2\theta_L}\right)^2 + (\theta_L^*)^2 + 2\theta_L^*$$

If topic is neutral or $\theta_E = 0$, then $\frac{dV_L}{d\lambda_L^*} = (\theta_L^*)^2 + 2\theta_L^*$. Now, suppose $\theta_E = q \in \mathbb{R}_{++}$ or a pro-right topic but not an extreme one, or $q << 1$. Then $\frac{dV_L}{d\lambda_L^*}$ is U-shaped. As $\lambda_L \to 0$, the fraction $\frac{\theta_E}{\theta_L}$ is negative. Now as λ_L increases such that $|\theta_L^*|$ decreases, then $\frac{\theta_E}{\theta_L}$ becomes more negative until λ_L increases enough to make L locate on the positive part of the ideology axis. Therefore, for ideologically negative issues, the maximum value function is U-shaped.

At the above threshold, the derivative of the maximum value function with respect to the equilibrium editorial choice vanishes.

10.5 Comparative statics in unbalanced reader population

$$\frac{d\theta_L^*}{d\lambda_L} = \frac{\theta_L^4 + \theta_L^3 + 0.25(\theta_E + \kappa)^2 - 0.5(\theta_E + \kappa)\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \tag{23}$$

$$\frac{d\theta_L^*}{db} = \frac{\frac{-c\theta_L^3(1-\theta_L)}{(b+(\theta_R-\theta_E)^2)^2}}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \tag{24}$$

$$\frac{d\theta_L^*}{d\kappa} = \frac{0.5\theta_L(\theta_E + \kappa) - 0.5\lambda_L\theta_L}{4\theta_L^3(1 - \lambda_L + \frac{c}{b+(\theta_R-\theta_E)^2}) + 3\theta_L^2(1 - \lambda_L - \frac{c\theta_E}{b+(\theta_R-\theta_E)^2}) + 0.5\lambda_L(\theta_E + \kappa)} \tag{25}$$

To elicit some important characteristics of media behavior, we first assume the parameters signifying the majority share and the cost of biasing through cross-over effect to be below some cutoff such that $|\kappa| < \hat{\kappa}$ and $b \geq \hat{b}$. This guarantees that the editorial choice are not shackled too much either by the extent of biased readers or by a very high cost of bias.

10.6 Model of a Monopoly news market

We do the similar analysis with only one partisan media serving the readers. Without loss of generality, let that firm be L situated at -1 on the ideology axis $[-1, 1]$. Symmetric results will hold if R is the monopoly firm.

This exercise is done to separate out the effects of competition for readership (in the absence of R) and learn the magnitude and direction of media slant due to ideology.

10.7 Utility Maximization of any reader i

This strategy of reader i is providing a rating to media L's report. Rating is a mapping from the ideology space to the real line $\mathbb{R}$, $\alpha_{iL} : \theta_L \to \mathbb{R}$.

$$U_i(\alpha_{iL}|\theta_L, \theta_E) = -(\alpha_{iL}\theta_L - x_i)^2 - (\alpha_{iL}\theta_L - \theta_E)^2 \tag{26}$$

The first term is quadratic loss in the distance between i's ideology x_i and the value (α_{iL}) which i attaches to the editorial position of L. The second term is similarly the distance between the weighted editorial position and the true signal θ_E from media E.

First Order Condition gives

$$\alpha^*_{iL} = \frac{x_i + \theta_E}{2\theta_L} = \frac{x_i + \theta_E}{\theta_L + \theta_L} \tag{27}$$

Second Order Condition for utility maximization is,

$$\frac{d^2U}{d\theta_L^2} = -4\theta_L^2 < 0 \tag{28}$$

10.8 Backward Induction by Media L

The action of the firm L is choosing a optimal editorial position $\theta^*_L \in [-1, 1]$, where

$$\theta^*_L = argmax_{\theta_L} \Pi_L(\theta_L)$$

Analogous interpretation holds for the optimal editorial stance θ^*_R of media R.

The payoff function of L is a quadratic loss function as shown below.

$$\Pi_L(\theta_L|\lambda_L, \theta_E) = -\lambda_L.(E(\alpha^*_L) - 1)^2 - (1 - \lambda_L)(\theta_L + 1)^2 - c(\theta_L - \theta_E)^2 \tag{29}$$

Equation (15) shows media L minimizes losses from two sources. θ^*_L minimizes the distance of reader i from attaining his best rating of 1 (given by $(E(\alpha^*_L) - 1)^2$). Simultaneously this choice

$\lambda_L \setminus \theta_E$	-1	0	1
0.1	-0.987	-0.45	0.296
0.5	-0.915	-0.312	0.453
0.9	-0.778	-0.083	0.635

Table 4: Equilibrium editorial position of media $L(\theta_L^*)$

also determines L's distance from its preferred ideology position of -1 (given by $(\theta_L+1)^2$).

The final term $c(\theta_L-\theta_E)^2$ denotes the convex cost of biasing news which is increasing with the distance of θ_L from the unbiased position θ_E. Moving farther away from the true signal θ_E require media to modify information more, thereby they incur higher cost. c denotes the marginal cost parameter with $c>1$.

First order condition of equation 6 gives:

$$\theta_L^4((1-\lambda_L)+c)+\theta_L^3((1-\lambda_L)-c\theta_E)+0.5\lambda_L\theta_E\theta_L-0.25\lambda_L(\theta_E)^2=0 \tag{30}$$

For clarity and ease of comparison with the previous sections we denote the equilibrium editorial choice of monopoly media L as θ_L^{M*}. [16].

Remark 8. *Comparing with proposition 1, θ_L^{M*} is always more biased towards the left than θ_L^*. The class of events supporting multiple equilibria is more biased to the right compared to the one in proposition 1.*

Table 4 illustrates the unique equilibrium values of monopoly media L which underscores corollary 2. .[17]

10.9 Comparative Statics

We devote this section to bring out subtle insights on how the parameter λ_L affects equilibrium strategy θ_L^*. In essence, we express θ_L^* as a function of λ_L.

Applying IFT on equation (7), we get,

$$\frac{\partial\theta_L}{\partial\lambda_L}=\frac{\theta_L^4+\theta_L^3+0.25\theta_E^2-0.5\theta_E\theta_L}{4\theta_L^3(1-\lambda_L+c)+3\theta_L^2(1-\lambda_L-c\theta_E)+0.5\lambda_L\theta_E} \tag{31}$$

The below proposition entails that moving away from the truth does pay the media with higher payoff upto threshold. For higher values of λ_L (media caring more about rating), then it will move closer to the truth.

[16] If R was the monopoly media, it would have been θ_R^{M*}

[17] To derive the numbers Table 4, we assume $c=1.1$ and $b=0.7$.

Remark 9. *There exists a threshold* $\lambda_L^* \in (0,1)$ *such that* $\frac{\partial \theta_L^*}{\partial \lambda_L} < 0$ *for* $\lambda_L \in (0, \lambda_L^*)$ *and* $\frac{\partial \theta_L^*}{\partial \lambda_L} > 0$, *for any* $\lambda_L > \lambda_L^*$.

The next proposition gives us a fair understanding of a comparison of the equilibrium profit levels of L due to a change in the values of the exogenous parameter λ_L. Once we express the equilibrium solution to the maximization problem in (6), we have θ_L^* as a function of λ_L and θ_E. If we substitute θ_L^* in the profit function, we obtain the maximum value profit function $V_L(.)$ of media L. This is a function of λ_L given a certain event characterized by θ_E. The variation in V_L due to changes in λ_L is a direct outcome of the envelope theorem.

10.10 Policy recommendation

As suggested by the educative role of media E instills a habit within readers to put greater weight on the true facts of the event which leads to the following characterization of news of media j by reader i.

$$\hat{\alpha_{ij}}^* = \frac{(2-\beta).x_i + \beta.\theta_E}{2\theta_j^*} \tag{32}$$

Initially, without such such educative role, this characterization is given by

$$\alpha_{ij}^* = \frac{x_i + \theta_E}{2\theta_j^*} \tag{33}$$

Policy recommendation matters when this role of media E increases welfare from reading news which is implied by the following.

$$-\sum_i [\hat{\alpha_{ij}}^* - 1]^2 > -\sum_i [\alpha_{ij}^* - 1]^2 \tag{34}$$

Expanding this leads to the following inequality

$$\begin{aligned} -(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N.\frac{\beta^2.\theta_E^2}{4\hat{\theta}_j^2} - \frac{2.(2-\beta).\beta.\theta_E.K}{4\hat{\theta}_j^2} - N + \frac{(2-\beta).K}{\hat{\theta}_j} + \frac{N.\beta.\theta_E}{\hat{\theta}_j} > \\ -\frac{\sum_i x_i^2}{4\theta_j^2} - N.\frac{\theta_E^2}{4\hat{\theta}_j^2} - \frac{2.\theta_E.K}{4\theta_j^2} - N + \frac{K}{\theta_j} + \frac{N.\theta_E}{\theta_j} \end{aligned} \tag{35}$$

where $\sum_i x_i = K$ denotes the polarization level.

10.10.1 When the event is neutral ($\theta_E = 0$)

Given $\theta_E = 0$, equation 35 is reduced to

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N + \frac{(2-\beta).K}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N + \frac{K}{\hat{\theta}_j}$$

or,

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} + \frac{(2-\beta).K}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} + \frac{K}{\theta_j} \tag{36}$$

The first terms on either side resembles the loss in utility due to increased spread of readers around the point 0, denoted by $\sum_i x_i^2$. The introduction of the new role of θ_E assuages the loss by a factor $0 < (2-\beta)^2 < 1$, given $\beta > 1$. Given $\theta_E = 0$, both θ_L and $\hat{\theta_L}$ are negative while θ_R and $\hat{\theta_R}$ are positive.

- When $K = 0$, then the new policy is effective iff the following holds

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} > -\frac{\sum_i x_i^2}{4\theta_j^2} \tag{37}$$

- When $K = -1$, then the new policy is effective iff the following holds

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - \frac{(2-\beta)}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} - \frac{1}{\theta_j} \tag{38}$$

- When $K = 1$

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} + \frac{(2-\beta)}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} + \frac{1}{\theta_j} \tag{39}$$

10.10.2 When event favours the left ($\theta_E = -1$)

When the event favours the left, overall reader utility can increase if the following inequality holds (follows from equation 35).

$$\begin{aligned} -(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N.\frac{\beta^2}{4\hat{\theta}_j^2} + \frac{2.(2-\beta).\beta.K}{4\hat{\theta}_j^2} - N - \frac{(2-\beta).K}{\hat{\theta}_j} - \frac{N.\beta}{\hat{\theta}_j} > \\ -\frac{\sum_i x_i^2}{4\theta_j^2} - N.\frac{1}{4\theta_j^2} + \frac{2.K}{4\theta_j^2} - N + \frac{K}{\theta_j} - \frac{N}{\theta_j} \end{aligned} \tag{40}$$

- When $K = 0$, then the new policy is effective iff the following holds

$$-(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N.\frac{\beta^2}{4\hat{\theta}_j^2} - \frac{N.\beta}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} - N.\frac{1}{4\theta_j^2} - \frac{N}{\theta_j} \tag{41}$$

Upon simplifying terms,

$$\frac{\sum_i x_i^2}{4}\Big[\frac{1}{\theta_j^2} - \frac{(2-\beta)^2}{\hat{\theta_j}^2}\Big] > -N\Big[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j}\Big] + 0.25N\Big[\frac{\beta^2}{\hat{\theta_j}^2} - \frac{1}{\theta_j^2}\Big] \tag{42}$$

- When $K = -1$, then the new policy is effective iff the following holds

$$\begin{split} -(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N.\frac{\beta^2}{4\hat{\theta}_j^2} - \frac{2.(2-\beta).\beta}{4\hat{\theta}_j^2} + \frac{(2-\beta)}{\hat{\theta}_j} - \frac{N.\beta}{\hat{\theta}_j} > -\frac{\sum_i x_i^2}{4\theta_j^2} - N.\frac{1}{4\theta_j^2} \\ -\frac{2}{4\theta_j^2} - \frac{1}{\theta_j} - \frac{N}{\theta_j} \end{split} \tag{43}$$

Simplifying terms

$$\begin{split} \frac{\sum_i x_i^2}{4}\Big[\frac{1}{\theta_j^2} - \frac{(2-\beta)^2}{\hat{\theta_j}^2}\Big] + 0.5\Big[\frac{1}{\theta_j^2} - \frac{(2-\beta)\beta}{\hat{\theta}_j^2}\Big] > N\Big[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j}\Big] + 0.25N\Big[\frac{\beta^2}{\hat{\theta_j}^2} - \\ \frac{1}{\theta_j^2}\Big] - \Big[\frac{1}{\theta_j} + \frac{2-\beta}{\hat{\theta}_j}\Big] \end{split} \tag{44}$$

- When $K = 1$, then the new policy is effective iff the following holds

$$\begin{split} -(2-\beta)^2.\frac{\sum_i x_i^2}{4\hat{\theta}_j^2} - N.\frac{\beta^2}{4\hat{\theta}_j^2} + \frac{2.(2-\beta).\beta}{4\hat{\theta}_j^2} - \frac{(2-\beta)}{\hat{\theta}_j} - \frac{N.\beta}{\hat{\theta}_j} > \\ -\frac{\sum_i x_i^2}{4\theta_j^2} - N.\frac{1}{4\theta_j^2} + \frac{2}{4\theta_j^2} + \frac{1}{\theta_j} - \frac{N}{\theta_j} \end{split} \tag{45}$$

$$\begin{split} \frac{\sum_i x_i^2}{4}\Big[\frac{1}{\theta_j^2} - \frac{(2-\beta)^2}{\hat{\theta_j}^2}\Big] - 0.5\Big[\frac{1}{\hat{\theta}_j^2} - \frac{(2-\beta)\beta}{\hat{\theta}_j^2}\Big] > -N\Big[\frac{\beta}{\hat{\theta}_j} - \frac{1}{\theta_j}\Big] + \\ 0.25N\Big[\frac{\beta^2}{\hat{\theta_j}^2} - \frac{1}{\theta_j^2}\Big] + \Big[\frac{1}{\theta_j} + \frac{2-\beta}{\hat{\theta}_j}\Big] \end{split} \tag{46}$$