Macroeconomics Qualifier Examination

June 2025

Suggested time to allocate: 1 hour per question.

Question:

Consider a centrally planned economy where households have a logarithmic utility function:

$$U = \int_0^{\infty} e^{-\rho t} \, lnC(t)dt$$

The central planner makes all allocation decisions and seeks to maximize the utility of households. There is L number of households in the economy, and each household has one unit of labor endowment. Assume that the output (final good) in this economy is produced according to

$$Y = L_y^{1-\alpha} \int_0^N x_j^{\alpha} dj$$

Where x_j represents intermediate good of variety j. The variable $L_y = \lambda L$ represents the part of the total labor supply (L) that is used in the final good production. The rest of the labor supply, $(L - L_y)$ are used to innovate new varieties of intermediate goods. Let $\mathrm{N}(t)$ denotes the number of varieties of intermediate goods available at time t. The number of new varieties of intermediate goods evolves according to

$$\frac{\dot{N}}{N} = \delta(L - L_y)$$

Where $\delta > 0$ is a given constant. At each moment in time, the central planner employs all available varieties of intermediate goods in the production of output, with each intermediate good used in equal amounts. Assume that one unit of an existing (one for which a blueprint is available) intermediate good requires one unit of final good (output) to produce.

(a) Set up the central planner's optimization problem while clearly outlining the set of choice variables and the set of constraints facing the central planner.
(b) Solve the central planner's problem to find the optimal growth rate of consumption.
(c) Finally, assume that $\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{N}}{N}$ holds for this economy. Use this condition to show that the central planner will always choose to allocate a fixed amount of labor, L_y in the production of the final good.

Question: Distinguishing Productivity and Cost-Push Shocks (45 minutes)

Consider a log-linearized New Keynesian model:

$$y_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{y_{t+1}\},$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^n) + u_t,$$

where output is produced using labor $y_t = a_t + n_t$ and productivity a_t is a zero-mean white noise. The cost-push shock is also a zero-mean white noise that is uncorrelated with a_t. The natural level of output is driven by productivity only $y_t^n = \psi a_t$, with $\psi > 0$. Finally, monetary policy follows a Taylor rule $i_t = \rho + \phi_\pi \pi_t$, with $\phi_\pi > 1$.

a) Use the method of undetermined coefficients and solve for y_t, π_t, n_tin terms of the two shocks a_t, u_t.

b) Suppose the central bank increases the coefficient ϕ_π. How does it affect the variance of y_t and π_t? How about the variance of the output gap $y_t - y_t^n$?

c) Suppose the data show that output y_t and employment n_t are positively correlated in the model, can we tell which shock is more important?

d) Suppose the data show that output y_t and employment n_t are negatively correlated. Based on the model, can we tell which shock is more important? What can we conclude about the parameter ψ?

Question: RBC model

Suppose the household preferences at period t is,

$$\ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi}$$

It earns a wage rate w_t. It can purchase one-period bond B_t at the price of q_t that matures at period $t+1$ (i.e. one unit bond costs q_t, which pays \$1 at period $t+1$). Households take bond price q_t as given.

Households also own the capital stock. There is a capital adjustment cost. Capital accumulates according to

$$K_{t+1} = \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1-\delta)K_t$$

where δ is the capital depreciation rate and I_t is investment.

Firms produces following a production function with labor N_t, capital K_t, and final goods X_t, and $\alpha + \beta + \gamma = 1$,

$$Y_t = A_t K_t^{\alpha} N_t^{\beta} X_t^{\gamma}$$

Thus, this is a roundabout economy, part of the final product is used for production.

Technology A_t is exogenous, following

$$\ln A_t = \rho \ln A_{t-1} + s_A \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1)$$

Set price of the good as numeraire.

a) Write down household's problem

b) Derive the first order conditions for the household problem.

c) Show that without capital adjustment cost, the ratio of

$$\frac{\text{marginal utility of having extra installed capital}}{\text{marginal utility of consumption}}$$

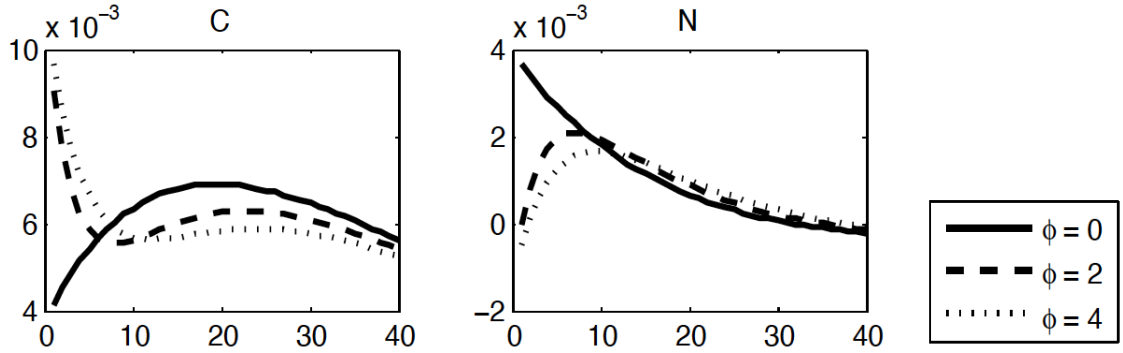
equals 1.

d) Write down firm's problem

e) Derive the first order condition for the final good used in production.

f) Write down the market clearing condition

g) The following graph plots the IRF after a positive TFP shock



Explain the intuition of the dynamics.