Department of Economics Qualifying Exam

Microeconomics

June 10, 2025

You are required to solve each of the 4 problems.
Note the time suggested for each problem.
Total duration: 3 hours.

Problem 1 (50 minutes)

Consider a pure-exchange economy with 2 units of good x and 2 units of good y. Agents A and B are each endowed with 1 unit of the respective goods. Determine the Walrasian equilibrium (give the allocation and the price ratio) for each of the two cases where the agents' utility functions are:

1. $u_A(x_A, y_A) = x_A + \alpha \ln y_A$, $u_B(x_B, y_B) = x_B + \beta \ln y_B$, with $\alpha, \beta \in (0, 1)$;

2. $u_A(x_A, y_A) = \sqrt{x_A} + \sqrt{y_A}$, $u_B(x_B, y_B) = x_B y_B$.

Problem 2 (40 minutes)

An investor has wealth w to be allocated between a risky asset and a riskless one. For each dollar invested in the risky asset, the gross return is either z_1 (with probability p) or z_2 (with probability $1 - p$). On the other hand, the riskless asset has a gross return of 1 for every dollar invested. The parameters z_1, z_2, p satisfy the following inequalities:

$$(i)\ 0 < z_1 < 1 < z_2,\ (ii)\ 1 < \frac{(1-p)(z_2-1)}{p(1-z_1)} < \frac{z_2}{z_1}.$$

Let θ denote the fraction of wealth allocated to the risky asset, with the remaining fraction $1 - \theta$ allocated to the riskless asset. Assume that the investor is an expected utility maximizer, with a utility function given by $u(x) = \ln x$ (for any level of sure income $x > 0$).

(a) Write the portfolio allocation problem and carefully argue that this agent will invest a strictly positive fraction of wealth in each of the two assets (that is to say, $0 < \theta < 1$).

(b) Explain why this allocation problem has a unique solution, and then compute the optimal fraction of wealth that should be invested in the risky asset.

Problem 3 (50 minutes)

Suppose that a union is the sole supplier of labor to all firms in a given oligopoly (such as the United Auto Workers is to General Motors, Ford, and others). The inverse demand (giving the market-clearing price as a function of aggregate output) is

$$P(Q) = \begin{cases} \delta - Q, & \text{if } Q \leq \delta; \\ 0 & \text{otherwise.} \end{cases}$$

To keep things simple, assume that the firms have no costs other than wages; and the production function for every firm $i \in \{1, \ldots, n\}$ is $q_i = L_i$ (that is, output equals labor). Hence, we also have $Q = L$, where $L = L_1 + \ldots + L_n$ stands for total employment in the unionized firms.

The dynamic game between the union and the firms is described as follows.

1. The union makes a wage demand, w, which applies to all firms.
2. The firms observe (and accept) w and then simultaneously choose their employment levels, L_i for each firm $i \in \{1, \ldots, n\}$.
3. The payoffs are determined: the union's utility is $(w - w_a)L$, where w_a is the wage that union members can earn in alternative employment; each firm i's profit is given by $(P(L) - w)\, L_i$.

(a) Use backward induction to derive the subgame perfect outcome of this game (assume that $0 \leq w_a < \delta$).

(b) Compute the union's utility under the subgame perfect outcome. How does it change with the number of firms? Interpret this result.

(c) Compute the equilibrium profit of each firm. How does this individual profit vary with the union members' alternative wage (w_a)?

Problem 4 (40 minutes)

Consider a Principal-Agent model of hidden action with **moving support** for the distribution of outputs. If the Agent deviates from the first-best level of effort, then the Principal observes (with positive probability) at least one output level that would not be possible if the first-best effort was exerted. By punishing the Agent severely for such an output realization, the Principal may be able to implement the first-best efficient risk-sharing solution despite the presence of moral hazard.

The Agent's utility, which is a function of wage w and effort e, is given by $v(w, e) = \sqrt{w} - e$, where e can be either 0 (low) or 1 (high). The Agent's reservation utility is $\overline{v} = 1$. The Principal is risk-neutral and has the utility $u(q, w) = q - w$, which is a function of output q and wage w. The Agent can produce different levels of output for the Principal with probabilities that depend on effort; and the Principal pays the Agent a wage that depends on output.

For $e = 0$, the output and corresponding probabilities are:

Output, q	$q = 0$	$q = 64$	$q = 100$
Probability	0.6	0.4	0

For $e = 1$, the output and corresponding probabilities are:

Output, q	$q = 0$	$q = 64$	$q = 100$
Probability	0	0.5	0.5

Note that the support of the output distribution changes from $\{0, 64\}$ under $e = 0$ to $\{64, 100\}$ under $e = 1$.

(a) Assuming first that the Principal observes the Agent's effort, compute the first-best outcome.

(b) Suppose now that the Principal cannot observe the Agent's effort. Show that the Principal can implement the first-best efficient outcome.