Macroeconomics Qualifier Examination

June 2024

Time allocated: 150 minutes.

Question 1:

Consider an economy with two sectors, a final good producing sector and an R&D sector that produces stock of knowledge. Fraction a_L of the labor force is used in the R&D sector, and the fraction $(1 - a_L)$ in the final goods-producing sector. Similarly, a fraction a_K of the capital stock is used in R&D and the rest in the final goods production. Accordingly, the quantity of output produced at time t is given by:

$$Y_t = [(1 - a_k)K_t]^{\alpha}[A_t(1 - a_L)L_t]^{(1-\alpha)}; \quad 0 < \alpha < 1$$

Y_t: The amount of final good produced at time t.

K_t: Capital Stock at time t.

L_t: The size of the labor force at time t.

A_t: The stock of knowledge at time t that augments labor productivity.

The production of new ideas depends on the quantities of capital and labor engaged in research and on the existing level of knowledge:

$$\dot{A}_t = B[a_K K_t]^{\beta}[a_L L_t]^{\gamma} A_t^{\theta}; \;\; B > 0, \;\; \beta \geq 0, \;\; \gamma \geq 0.$$

As in the Solow model, the savings rate is exogenous and constant. In addition, depreciation is set to zero. Thus,

$$\dot{K}_t = sY_t.$$

Finally, the population grows at an exogenous rate $n > 0$. Therefore,

$$\dot{L}_t = nL_t$$

Define $g_K(t)$ and $g_A(t)$ as the growth rates of K and A at time period t.

(i) Note that both $g_K(t)$ and $g_A(t)$ are time variant. Find the combinations of $g_K(t)$ and $g_A(t)$ for which $g_K(t)$ will remain constant (i.e., $\dot{g}_K = 0$).

(ii) Find the combinations of $g_K(t)$ and $g_A(t)$ for which $g_A(t)$ will remain constant (i.e., $\dot{g}_A = 0$).

(iii) Derive the condition under which the economy will converge to a state where both K and A will continue to grow at constant rates. Find explicit expressions for these rates.

(iv) How do the above results parallel those from any benchmark model in the modern growth literature?

Question 2:

1. Consider the following two-period OLG model. Time is discrete and indexed by t = 0, 1, 2, ...,. In period t there are L_t two-period-lived agents are born, where $L_t = (1+n)^t L_0$, with L_0 given and $n > 0$. At $t = 0$ there are some one-period-lived old agents who are the owners of the initial capital stock, K_0. A young agent in period t has preferences given by $u\left(c_t^y, c_{t+1}^o\right) = lnc_t^y + \beta lnc_{t+1}^o$, where the superscripts 'y' and 'o' are meant to represent young and old age consumptions. Each agent has one unit of labor available when young, which they supply inelastically to the firms for competitively determined wage rates. The production technology is given by $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where Y_t is output, K_t is the capital input, L_t is the labor input, A is a technological constant, and 0 < α < 1. One unit of the consumption good can be converted into one unit of capital, and vice-versa.
   (a) Determine the steady-state capital/labor ratio for the decentralized economy.
   (b) Determine the socially optimal steady-state capital-labor ratio that maximizes per-capita consumption at the steady state.
   (c) Suppose that the government issues B_{t+1} amount of one period bond in period t. These bonds mature in period $(t+1)$ and pay $B_{t+1}(1+r_{t+1})$ units of consumption goods in the period $(t+1)$. Assume that it is mandatory for each young agent at time t to buy b amount of bond, so that $B_{t+1} = bL_t$. Also, assume that the government levies a lump-sum tax τ_t on each young agent so that total tax revenue $T_t = \tau_t L_t$. Finally, assume that the government balances its budget every period.

      (i) Write down the government budget constraint for period t.
      (ii) When the government is present, find the law of motion for the capital-labor ratio (i.e., the equation connecting k_{t+1} and k_t).
      (iii) Is the socially optimum steady-state is different from the steady-state for a decentralized economy with government present? If yes, then find the value of b which a central planner can choose to induce a socially optimum steady state.