Macroeconomics Qualifier Examination

August 2024

Time allocated: 150 minutes.

Question 1:

Consider an economy where the aggregate production function is given by

$Y = F(K, L, Z) = L^{\beta} K^{\alpha} Z^{1-\alpha-\beta}$. Here, L and K denote aggregate labor and capital, respectively. Z denotes the aggregate amount of a resource (e.g., land) that is fixed in supply. Assume that $\alpha + \beta < 1$. The economy saves s fraction of the output (Y). Capital depreciates at a rate δ.

(a) Suppose that the population size is constant. Find the steady-state capital-labor ratio and the per capita steady-state output.

(b) Show that the steady state of the capital-labor ratio is unique and is globally stable.

(c) Suppose that the population grows at a constant rate n. What happens to the steady state capital-labor ratio as $t \to \infty$?

Question 2:

Consider an economy with zero population growth. A central planner makes the decision on behalf of representative agents and wishes to maximize $\sum_{t=0}^{\infty} \beta^t \frac{c_t^{\gamma}}{\gamma}$, where c_t is the consumption per capita, β represents the discount factor, and $\gamma < 1$. Each member of the population is endowed with one unit of labor. The central planner makes the decision about how to allocate an agent's 1 unit labor endowment between the production of current output and the accumulation of human capital, h. Each individual can produce output using $y_t = \alpha h_t u_t$, where h_t is the amount of human capital available to an individual. The fraction of labor endowment allocated to output production is denoted by u_t, and $\alpha > 0$ represents a technological parameter. Human capital is produced using the technology $h_{t+1} = \delta h_t (1 - u_t)$, where $\delta > 0$ and $(1 - u_t)$ denotes the fraction of time t labor endowment allocated toward the accumulation of human capital. Assume that the initial stock of human capital, h_0, is given. Finally, assume that each individual consumes the entire output, i.e., $c_t = y_t = \alpha h_t u_t$.

a. Set up the optimization problem for the central planner.
b. Use Bellman Equation to solve the problem and derive the Euler equation for consumption.
c. Derive the growth rate of consumption, $\frac{c_{t+1}}{c_t}$.

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d. How u_{t+1} and u_t are related along the optimal path? What is the steady state value of u?

Question 3:

Consider a typical Diamond model where individuals only live for two periods. At time t, each young individual supplies one unit of labor to the market to earn a market-determined wage rate, w_t. In addition, each young individual receives an inheritance (b_t) from his/her parents. Suppose that individuals are only interested in old-age consumption. As a result, the entire wage income, w_t, and inheritance, b_t, are saved during the first period of life. Savings become the capital source for the next period and earn a fixed gross rate of return r. During adulthood, an individual needs to decide how to divide the available fund, $(w_t + b_t) * r$, between old age consumption, c_{t+1}, and bequests, b_{t+1}. Assume that wage rate, w_t, is linear in time t capital-labor ratio: $w_t = \Omega k_t$, where $\Omega < 1$ is a constant. All agents have identical preferences defined over consumption and bequests. An agent born at time t derives lifetime utility, U_t, according to

$$U_t = c_{t+1} + \Phi(b_{t+1})$$

Where c_{t+1} denotes old age consumption, b_{t+1} denotes bequests and $\Phi(.)$ is monotonically increasing in bequest amount with properties $\Phi' > 0; \Phi'' < 0$. Finally, assume that the population growth rate is zero.

(i) Find the optimal levels of consumption, c_{t+1}, and bequests, b_{t+1}.
(ii) What is the relationship between b_t and b_{t+1}?
(iii) Find the steady-state capital-labor ratio for this economy.