**Macroeconomics Qualifier Examination**

**August 2023**

**Time allocated: 120 minutes.**

**Question 1:**

Consider an economy with zero population growth where the output $(Y_t)$ is produced according to the production function $Y_t = K_t^{1-\alpha} H_t^{\alpha}$. Here, $H_t$ and $K_t$ denote human and physical capital, respectively. Suppose that the economy allocates $s_k$ and $s_h$ fractions of the current output toward the accumulation of physical capital and human capital, respectively. Also, assume that physical and human capital depreciates at a rate $\delta$.

a) Derive the equations dictating the growth rates of physical and human capital.

b) Suppose that along the growth path, the economy maintains the condition that the marginal product of physical capital equals the marginal product of human capital. Use this condition to derive the ratio of physical to human capital that needs to be maintained throughout the growth path.

c) What is the ratio of physical to human capital at the economy's steady state?

d) Use the above results to show that the production function of the economy can be rewritten as an $AK$ production function.

**Question 2:**

Consider a typical Diamond overlapping generation model where individuals only live for two periods. Each young individual supplies one unit of labor when young, allocating labor income between the first-period consumption and savings. During old age, individuals consume the savings and any interest income on savings. Let $C_{1t}$ and $C_{2t}$ denote the consumption in the period $t$ of young and old individuals. The utility of an individual born at the time $t$ is given by

$U_t = \log C_{1t} + \dfrac{1}{1+\rho} \log C_{2,t+1}$. There are many output-producing firms, each of which

produces output according to the production function $y_t = k_t^\alpha$, where $y_t$ is the output per

worker and $k_t$ denotes the capital-labor ratio. Finally, assume that the population grows at an

exogenously given rate $n$.


(i)     Find the steady-state capital-labor ratio and output-labor ratios for this economy.

(ii)    Suppose the government taxes each young individual an amount $T$ and uses the proceeds
        to pay benefits to old individuals. Demonstrate how, if at all, does this change the capital-
        labor ratio at the steady state?