Department of Economics Qualifying Exam

Microeconomics

July 31 2024

Please answer all 4 questions
Notice the time allocated to each question.

Problem 1: (40 Minutes) Consider a consumer who has a set of preferences over the bundle of goods (x, y) represented by the utility function $u(x, y)$. Suppose further that the indifference curves of this utility function for an arbitrary positive level of utility k are given by the following expression

$$\begin{cases} y + 2x = k & \text{if } x < \frac{k}{3} \\ 2y + x = k & \text{if } x \geq \frac{k}{3} \end{cases}$$

(a). Plot the indifference curve for some positive value of k

(b). Given arbitrary positive prices p_x, p_y and positive income m derive the Marshallian demands for this consumer.

Problem 2: (50 Minutes) Consider an auto manufacturer who is facing a choice between producing regular or hybrid cars for the next period. The profit that it can earn from either type depends on whether gas prices will be high or low next period. Let $0 < p < 1$ be the probability that gas prices will be high. The profit per car is given by:

	High Gas Prices	Low Gas Prices
Hybrid	\$36	\$1
Regular	\$9	\$25

The manufacturer plans to produce 100 cars next year. The utility function of the manufacturer is $u(\pi) = \sqrt{\pi}$, where π are the profits.

(a) Suppose the manufacturer plans to produce cars of only one type. For what values of p would the manufacturer produce only hybrid cars?

(b) Suppose $p = \frac{1}{2}$ and the manufacturer plans to produce 50 hybrid cars and 50 regular cars. Is this better than producing all regular cars or all hybrid cars?

(c) Suppose $p = \frac{1}{2}$. Let α denote the number of hybrids that are produced (and $100 - \alpha$ the number of regular cars), where $\alpha \in [0, 100]$. The manufactures chooses α to maximize expected utility. Derive the optimal value of α. What is the expected utility from this optimal mix?

Problem 3: (60 Minutes) For the following consider a pure exchange economy with two consumers $I = \{A\, B\}$ and two goods x, y. It is assumed that total endowments are $\bar{x} = 100$, $\bar{y} = 100$ with $\bar{x}_A = \bar{x}_B = 50$ and $\bar{y}_A = \bar{y}_B = 50$. Find the equilibrium allocation and represent it via an Edgeworth box when consumers have the the following utilities:

(a). $u_A(x_A, y_A) = x_A$, and $u_B(x_B, y_B) = y_B$

(b). For $i \in \{A, B\}$, $u_i(x_i, y_i) = min\{\frac{x_i}{2}, y_i\}$

(c). $u_A(x_A, y_A) = min\{\frac{x_A}{2}, y_A\}$, and $u_B(x_B, y_B) = min\{x_B, \frac{y_B}{2}\}$

Problem 4: (50 Minutes) Consider a principal-agent model with a risk-neutral principal and a risk-averse agent. The agent has a utility function, $v(w, e) = \sqrt{w} - e$, where e can be 0 or 1. The agent's reservation utility is equal to 5. The agent can produce either 0 or 100 units of output, q, with probabilities that depend on effort:

Effort, e	Probability of output	
	$q = 0$	$q = 100$
$e = 0$	0.4	0.6
$e = 1$	0.2	0.8

A. Derive the optimal contract when the principal can observe the agent's effort (first-best). What are the (expected) profits of the principal?

B. Derive the optimal contract when the agent's effort is hidden from the principal (second-best). What are the (expected) profits of the principal? How do (expected) profits for the principal compare in the first-best and second-best cases?