Departments of Economics Ph.D. Qualifying Exam June 2024

PART I: MICRO

June 4th, 2024

Please answer all 4 questions.
Notice the time allotted to each question.

Problem 1 (50 minutes)

Determine the Marshallian demands in the case of the following utility functions for $(x, y) \in \mathbb{R}^2_+$. Unfortunately, given the non-differentiability of the utility function, the Lagrangian technique is inapplicable in most cases. However, determining the utility maximizing choice graphically will help.

(a) $U(x, y) = \min\{x, y\}$.

(b) $U(x, y) = \max\{x, y\}$.

(c) $U(x, y) = x + 2y$.

(d) $U(x, y) = xy$.

Problem 2 (40 minutes)

Consider a pure exchange economy with two consumers (A and B) and two goods (X and Y). Suppose consumers A and B have the utility functions:

$$\begin{aligned} u^A(x_A, y_A) &= x_A^\alpha y_A^{1-\alpha}, \qquad 0 < \alpha < 1 \\ u^B(x_B, y_B) &= x_B^\beta y_B^{1-\beta}, \qquad 0 < \beta < 1 \end{aligned}$$

(a) Derive the equation of the contract curve when the aggregate endowment bundle is (ω_X, ω_Y).

(b) Let the endowments of the economy be 1 unit of X and 2 units of Y. Show that as a rule, an equal division of goods (an **egalitarian** outcome) is not Pareto optimal. What condition on the parameters of the utility functions is required for equal division of goods to be Pareto optimal?

Problem 3 (60 minutes)

A domestic firm (Firm 1) and a foreign firm (Firm 2) produce an identical product which they sell in a third country. (The third country assumption allows us to ignore consumer welfare in the computations.) The government in the domestic country understands the structure of the industry and offers the domestic firm an export subsidy in advance of the quantity decisions by the two firms. There are two stages in the game:

Stage 1: Government in domestic country sets a per unit export subsidy, s, to maximize the profits of the domestic firm net of the total subsidy.

Stage 2: The domestic and foreign firms observe the per unit export subsidy set by the domestic government. The two firms then play a Cournot game in a third country where they face the homogeneous market demand:

$$P = \alpha - Q_1 - Q_2, \quad \alpha > 0 \tag{1}$$

It is assumed for simplicity that the two firms have zero costs of production. The payoffs to the domestic and foreign firm are respectively:

$$\pi_1(Q_1, Q_2, s) = (\alpha - Q_1 - Q_2)\, Q_1 + sQ_1 \tag{2}$$

$$\pi_2(Q_1, Q_2) = (\alpha - Q_1 - Q_2)\, Q_2 \tag{3}$$

The objective of the domestic government is to choose the per unit export subsidy, s, to maximize its welfare function. The welfare function is defined as the profits of the domestic firm minus the cost of providing the subsidy:

$$W_1(Q_1, Q_2, s) = \pi_1(Q_1, Q_2, s) - sQ_1 = (\alpha - Q_1 - Q_2)\, Q_1$$

A. Given the per unit export subsidy s from the first period, compute the second-stage Cournot-Nash equilibrium. How do the Cournot-Nash quantities of the two firms vary with the level of the per unit export subsidy?

B. What is the per unit export subsidy set by the domestic government in the subgame perfect equilibrium? What are the outputs produced by the two firms in the subgame perfect equilibrium? What can you say about the subgame perfect equilibrium outputs? (**Hint:** Compare the outputs to a Stackelberg game with zero subsidy and with firm 1 as leader and firm 2 as follower).

Problem 4 (30 minutes)

The game is between a job applicant and an employer. Nature starts the game by determining the ability level, a of the applicant. The ability level can be High (a=4) with probability 1/2 and Low (a=1) with probability .5. The applicant can observe Nature's move and knows her/his type but the employer does not.

However, the applicant can choose a signal (level of education) $e = 0$ or 1. The cost of sending a signal e (achieving a level of education $e = 1$) is **1** for H and **4** for L . Therefore, signaling is costly and the cost is lower for the High ability applicant.

The employer observes the signal, e, sent by the applicant and offers a wage, $w(e)$, that depends on the signal e. If the applicant accepts, then the applicant produces output according to the production function, $y(a,\ e) = a$. The High ability applicant produces more output than the Low; further, education has no impact on output.

The payoff to the applicant is $w(e)$ - $C(a,\ e)$, while the profit of the employer is $y(a,\ e)$ - $w(e)$. Assume that the employer equates wage to output (thereby making zero profits).

1. Suppose the workers cannot signal (no schools available), what is the prevailing wage?

2. Is it possible for both types to choose $e = 1$? in equilibrium. Explain!

3. Is it possible to obtain an equilibrium in which the High ability workers choose $e = 1$ while the Low ability opts for $e = 0$? Explain!