Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam June 2023

PART I: MICRO

June 12, 2023

Please answer all 4 questions.
Notice the time allotted to each question.

<u>Problem 1</u> (50 minutes)

Let $\succsim_L$ denote the lexicographic preference relation on $\mathbb{R}^2_+$.

(i) Show that $\succsim_L$ does not have a utility function representation.

(ii) We know that preferences which have a utility representation are rational, that is complete and transitive.
Show that $\succsim_L$ is transitive, even though it does not have a utility representation.

(iii) For a consumer with preference relation $\succsim_L$, determine the Marshallian demand $x(p, m)$ at price system $p = (p_1, p_2) \gg 0$ and income $m > 0$.

(iv) For a consumer whose preferences on $\mathbb{R}^2_+$ are represented by $u(x_1, x_2) = x_1$, determine the Marshallian demand $x(p, m)$ at price system $p = (p_1, p_2) \gg 0$ and income $m > 0$.

(v) Can one infer a consumer's preferences from the consumer's demand correspondence? Compare your answers in (iii) and (iv)!

Problem 2 (50 minutes)

Felix Chance has current wealth $W > 0$. Felix invests the amount C with $0 \leq C \leq W$ in a risky asset and the amount $W - C$ in a safe asset.

(A) Investing the amount C in the risky asset yields future income

1) $z_1 C$ with probability $p_1 > 0$,

2) $z_2 C$ with probability $p_2 > 0$,

...

n) $z_n C$ with probability $p_n > 0$

where $n \geq 2$, $0 \leq z_1 < z_2 < \ldots < z_n$ and $\sum_{i=1}^{n} p_i = 1$. Let $z = \sum_{i=1}^{n} p_i z_i$ be the expected per unit return of the risky investment.

(B) Investing the amount $W - C$ in the safe asset yields future income $s(W - C)$ with $s > 0$.

Felix's future wealth consists of his investment income. He is an expected utility maximizer with respect to future wealth, with von Neumann-Morgenstern utility function $u(w)$ for future wealth $w \geq 0$. u is twice differentiable with $u' > 0$ and $u'' < 0$.

(a) Write down the expression for $EU(C)$, Felix's expected utility of future wealth as a function of C.

(b) Derive $EU'(C)$, the derivative of EU with respect to C.

(c) Derive $EU''(C)$.

(d) What is the sign of $EU''(C)$ if $z \neq s$?

(e) Determine the sign of $EU'(0)$ by comparing z and s.

(f) When is it optimal for Felix to choose a positive C, that is to take some risk although Felix is risk averse?
HINT. Given the answer in (e), when is $C = 0$ not optimal?

(g) Suppose $n = 2$, $z_1 = 0$, $z_2 = 1.8$, $p_1 = p_2 = 1/2$, $s = 0.8$, and $u(w) = \sqrt{w}$ for $w \geq 0$.
Determine the value of C that maximizes $EU(C)$.

Problem 3 (50 minutes)

Let the production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ be given as

$$f(z) = \begin{cases} z^2 & \text{for } z \leq 1; \\ 2\ln z + 1 & \text{for } z \geq 1. \end{cases}$$

(a) Sketch the corresponding technology Y in a diagram.

(b) Find z^o such that $f'(z)$ is maximized at z^o. Determine $f'(z^o)$.

(c) For all output prices $p > 0$ and input prices (factor prices) $w > 0$, determine the inputs at which the FOC (First-Order Condition)
$f'(z) = w/p$ for profit maximization is satisfied.
HINT. Distinguish the cases $w/p > 2, w/p = 2$ and $w/p < 2$.

(d) For all output prices $p > 0$ and input prices (factor prices) $w > 0$, calculate the firm's profit with the input(s) satisfying the FOC which you found in (c). When is the profit non-negative?

(e) Using the results in (c)and (d), solve the profit maximization problem

$$\max_{z \geq 0} pf(z) - wz$$

for all output prices $p > 0$ and input prices (factor prices) $w > 0$.

Problem 4 (60 minutes)

Two firms 1 and 2 with a unit production cost $c > 0$ compete in a product market with linear inverse demand function $P(Q) = a - Q$, where $a > c$. Firm 1 first decides whether to enter the market at a cost $F > 0$. In case firm 1 enters the market, firm 2 decides whether to compete against it à la Bertrand or Cournot. In case firm 1 stays out of the market, firm 2 is the monopoly firm and maximizes the monopoly profit.

(i) Formalize the situation as a normal-form game and solve for all Nash equilibria.

(ii) Formalize the situation as an extensive-form game and solve for all sub-game perfect equilibria.